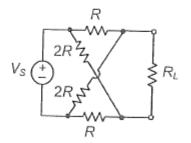
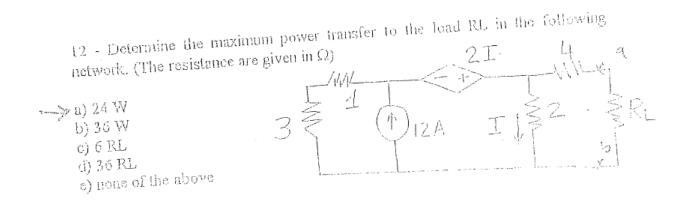
- Determine R_L for maximum power transfer, and calculate the value of this power, assuming V_S = 12 V and R = 3 Ω.
- →A. 4 Ω; 1 W B. 8 Ω; 0.5 W
 - C. 4Ω; 4W
 - D. 8Ω; 2W
 - E. None of the above



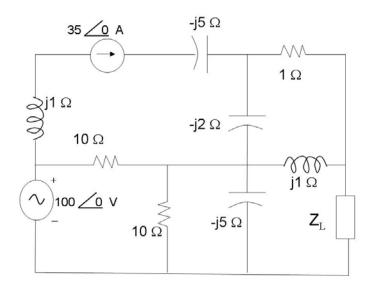


- 8. The source current in the circuit shown is 3 cos(5000t) A. What impedance should be connected across the terminals a,b for maximum average power transfer. 3.6 mH 108 a a 10-20 j Ω 20A 49 42 è. None of the above. C
- \rightarrow b 20-10 j Ω c. $10 + 20 j \Omega$. d. 10-20 j Ω.

3/4

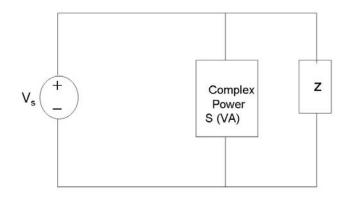
Problem 2

Find the value of the load impedance Z_L such that maximum real power is delivered to it



A) $3.0 + j1.0 \Omega$ B) $1.75 + j0.25 \Omega$ C) $1.5 - j0.5 \Omega$ D) $3.0 + j1.1 \Omega$ E) None of the above

Problem 16



It is given that the load above has complex power S==20 + 15j KVA. It is required to connect an element Z in parallel with the load so as to correct the power factor to unity (power factor= 1). The source voltage is $V_s=200 \angle 50$ V rms and the frequency is 60 Hz [that is $V_s=200\cos(377t+50^\circ)$ V (rms)]. Determine the value and nature of this element Z.

A) Capacitor with value C=994.7 μF
 B) Inductor with value L=7.07 mH
 C) Capacitor with value C=663.13 μF
 D) Inductor with value L=10.61 mH
 E) None of the above.

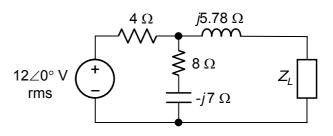
Problem 8 (14 pts)

Consider the circuit shown

a. Determine the value of the load Z_L to maximize the average power absorbed by the load. (4 pts)

$$Z_{src} = j5.78 + \frac{4(8 - j7)}{12 - j7}$$

= 3 + j5.2 Ω;
$$Z_{Lm} = 3 - j5.2 \Omega.$$



b. For the value obtained in (a), determine the average power developed by the voltage source and the average power absorbed by the load. (4 pts)

$$V_{Th} = 12 \frac{8 - j7}{12 - j7} = 9 - j1.74 \text{ V};$$

$$|V_{Th}| = 9.18 \text{ V};$$

$$I_{L} = V_{Th}/6 = 1.5 - j0.29 \text{ A}; \quad 12 \angle 0^{\circ} \text{ V}$$

$$V_{1} = (3 - j0.58)I_{L} = 4.68 + j0 \text{ V} \text{ rms}$$

$$I_{SRC} = \frac{12 - 4.68}{4} = 1.83 + j0 \text{ A}$$

$$P_{SRC} = V_{1}I_{SRC} = 12 \times 1.83 \cong 22 \text{ W}; P_{L} = \frac{(9.18)^{2}}{4 \times 3} \cong 7 \text{ W}.$$

c. For a purely resistive load, determine its value R_{max} for maximum power transfer and find the power absorbed by the load. (3 pts)

$$R_m = \sqrt{(3)^2 + (5.2)^2} \cong 6 \ \Omega; \ |\mathbf{I}| = |\mathbf{V}_{\mathsf{Th}}|/|Z| = \frac{9.18}{\sqrt{(3+6)^2 + (5.2)^2}} = 0.883 \ \mathsf{A}$$
$$P = |\mathbf{I}|^2 \times 6 = 4.68 \ \mathsf{W}.$$

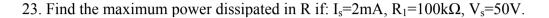
d. For a purely resistive load with $R_L = 2R_{max}$ (R_{max} of part c), determine the power absorbed by the load. (3 pts)

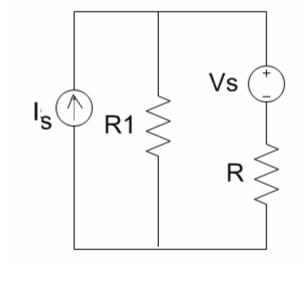
$$|\mathbf{I}| = |\mathbf{V}_{\mathsf{Th}}|/|Z| = \frac{9.18}{\sqrt{(3+12)^2 + (5.2)^2}} = 0.578 \text{ A}; P = |\mathbf{I}|^2 \times 12 \cong 4 \text{ W}$$

17. If a capacitor with impedance Z_2 is connected in parallel to a load $Z_1 = 300 + j450 \Omega$. What should be Z_2 in ohms so that the equivalent load is purely resistive?

18. What is the power factor of the equivalent load of the previous question?

a) 0.8 b) 0.6 c) 0 →d) 1 e) None of the above





a) P= 12.5 mW
b) P= 1.25 mW
c) P= 50 mW
→d) P= 56.25 mW
e) None of the above

-1- Two inductive loads of 0.88 KW and 1.32 KW at power factors of 0.8 and 0.6 respectively are connected in parallel across a 220-V (rms), 50Hz supply. Calculate the total current taken by this combination.

a. 1A_b. 14.86A c. 10.86A d. 15.45A e. None of the above

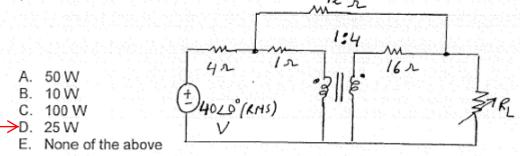
-2- For the previous problem, find the value of capacitance in microfarads, to be connected in parallel with the loads to bring the combined power factor to 0.9 lagging.

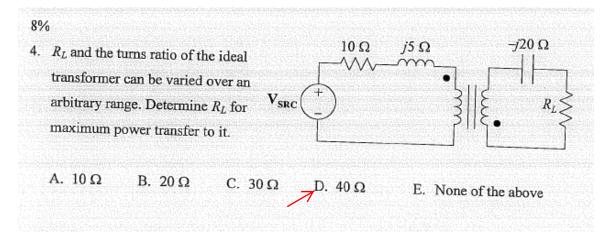
→a. 89 b. 35.8 c. 25.6 d. 44.5 e. None of the above

•6- Two impedances $Z=(2+j4) \Omega$ and $Z'=R \Omega$ are connected in parallel. Find R so that the power factor of the circuit is 0.9 lag.

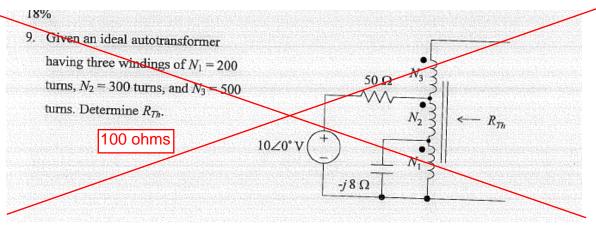
a. 1.3Ω b. 3.2Ω c. 2.4Ω d. 3Ω e. None of the above

Find the maximum average power given that R_L is adjusted for maximum power transfer.

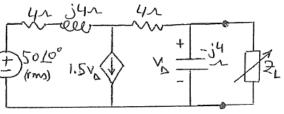




Solution: To have the reactances add to zero, the transformer turns ratio must be 2, primary-to-secondary. Hence $R_L = 4 \times 10 = 40 \Omega$.



-3- The load impedance Z_L for the circuit shown is adjusted until maximum average power is delivered to the load. Find this maximum power.

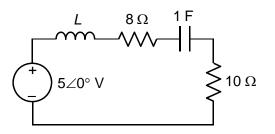


a. 5W b. 25.34W c. 296.8W d. 7.81W e. None of the above

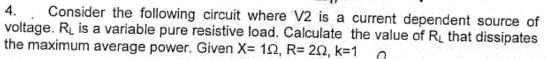
11. Determine the frequency at which maximum power is dissipated in the 10 Ω resistor, assuming *L* = 1 H. **Solution:** $\frac{1}{\omega C} = \frac{1}{\omega} \Omega$. Maximum power is

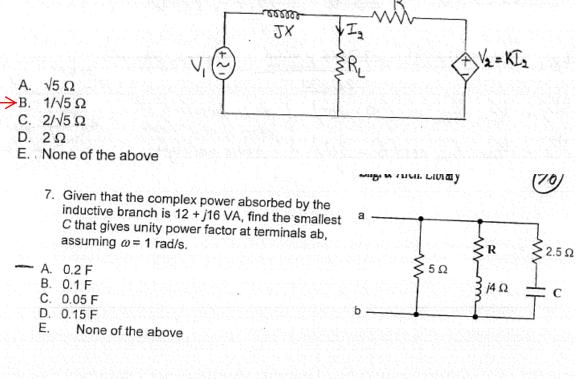
dissipated in the 10 Ω resistor when $X_L = -X_C$,

which gives
$$\omega L = \frac{1}{\omega}$$
, or $\omega = \frac{1}{\sqrt{L}}$ rad/s.

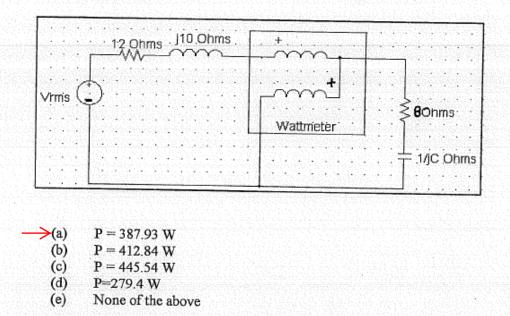


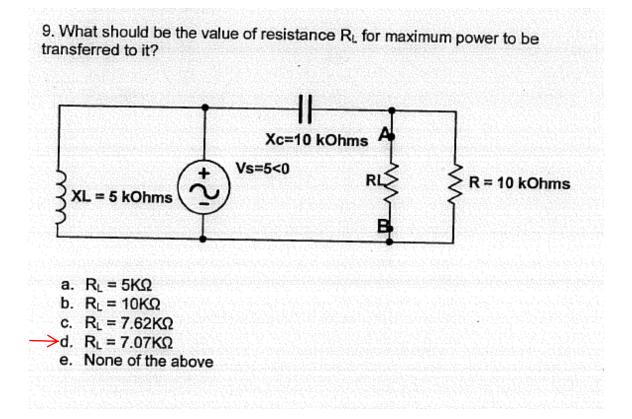
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18. Find the wattmeter reading of the circuit below. Where C = 1/2 F $V_{rms} = 150 < 0$, V





5. An electric motor draws an active power of 100 kW at 0.8 p.f lagging from a 240 V, 60 Hz source. This motor is connected in parallel to another load of $0.1 + j0.4 \Omega$. What is the size of the parallel connected capacitor needed to raise the total power factor to 0.95 lagging. a. 6.27 mF b. 4.25 mF - c. 7.68 mF d. 2.88 mF e. None of the above 100 KW 240V 0.1+ 10.4 0.8 Lag. (RMS) 60Hz

2. A 1 Ω resistor is connected in parallel with a d'Arsonval movement having a full scale deflection of 1 mA. If a 40 mA current produces a deflection that is 80% of full scale, determine the resistance of the d'Arsonval movement.

a) 58Ω
→b) 49Ω
c) 37Ω
d) 76Ω
e) None of the above

- 8. A 300-V voltmeter that draws 2mA current for full-scale reading is used to measure the voltage across the $50-K\Omega$ resistor of Figure 6. The voltmeter reading is:
 - 60V а.
 - b. 120V
 - 40V с.
 - 60 90V
 - None of the above е.



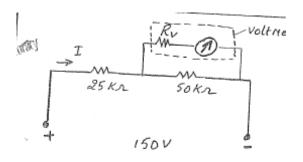
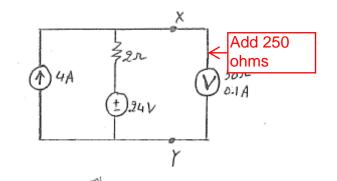
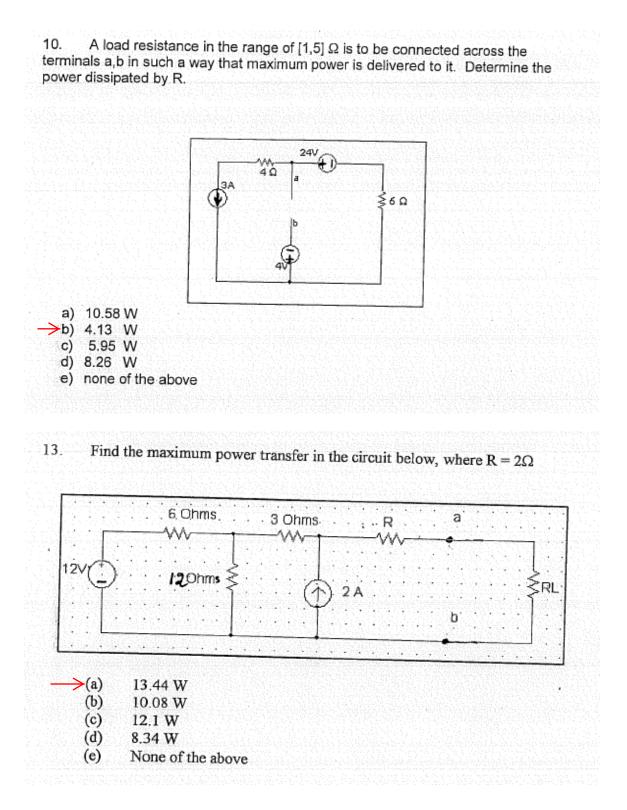


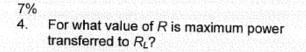
Fig.6

- 15. A 502, 0.1A d'Arsonval meter movement is used in a voltmeter circuit (Figure 13). Determine the voltmeter reading across the terminals x-y on a full-scale of 30V.
 - 26.49V 8.
 - b. 32V 31.79V
 - 36V d.
 - ē.

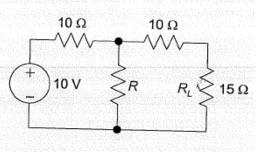
None of the above







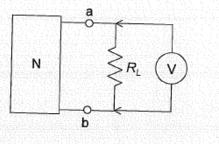
- A. 10Ω
- Β. 15 Ω
- C. 20 Ω
- D. Infinite resistance
 E. None of the above

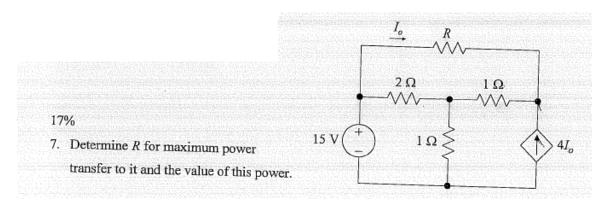


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5. A circuit N has an open-circuit voltage of 15 V between terminals ab, and an unknown source resistance R_s . A voltmeter across ab reads 12 V when $R_L = 10 \text{ k}\Omega$ and 10 V when $R_L = 40/9 \text{ k}\Omega$. Determine R_s and R_v , the resistance of the voltmeter.

A. $R_S = 2 \text{ k}\Omega$, $R_V = 40 \text{ k}\Omega$ B. $R_S = 2 \text{ k}\Omega$, $R_V = 80 \text{ k}\Omega$ C. $R_S = 4 \text{ k}\Omega$, $R_V = 40 \text{ k}\Omega$ D. $R_S = 4 \text{ k}\Omega$, $R_V = 80 \text{ k}\Omega$ E. None of the above

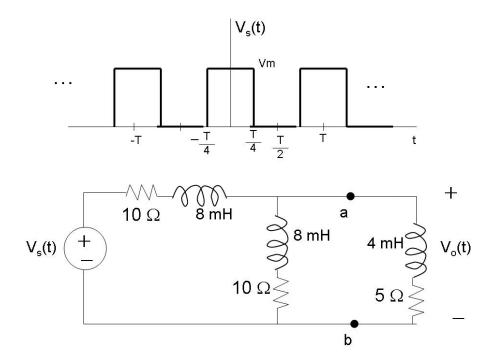




Solution: When R is replaced by an open 2Ω 1Ω circuit, the current source is set to zero. The circuit becomes as shown. $V_{Th} = V_{ab} =$ 15 V 1Ω $15 \times \frac{20}{30} = 10$ V. When a source V_T is applied, with the V_T 15 V short circuited, and IT as shown, the polarity of the current source is reversed. From KVL: $V_T = 5I_T$ (1+2||1). This gives $\frac{V_T}{I_T} = R = R_{Th} = \frac{25}{3} \Omega$. Max 2Ω $1 \Omega 5I_T$ power transferred is $\frac{100}{4 \times 25/3} = 3$ W. $1\,\Omega$ $4I_T$

Problem 3

Consider the periodic signal $V_s(t)$ shown below. Assume this signal is applied to the circuit in the figure below, find an expression for the voltage $V_o(t)$ across the terminals a,b as shown.



$$\rightarrow$$
 A) $V_o(t) = \frac{V_m}{8} + \frac{V_m}{2\pi} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{2}}{n} \cos n\omega_o t$

B)
$$V_0(t) = \frac{V_m}{8} + \frac{V_m}{4\pi} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{4}}{n} \cos n\omega_o t$$

C)
$$V_4(t) = \frac{V_m}{4} + \frac{V_m}{2\pi} \sum_{n=1}^{\infty} \frac{\sin\frac{n\pi}{2}}{n} \cos n\omega_o t$$

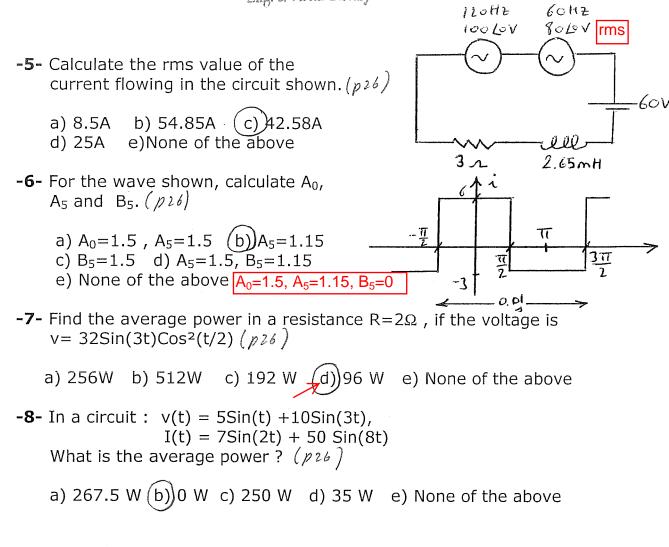
D)
$$V_4(t) = \frac{V_m}{4} + \frac{V_m}{2\pi} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{2}}{n} \cos n\omega_o t$$

E) None of the above

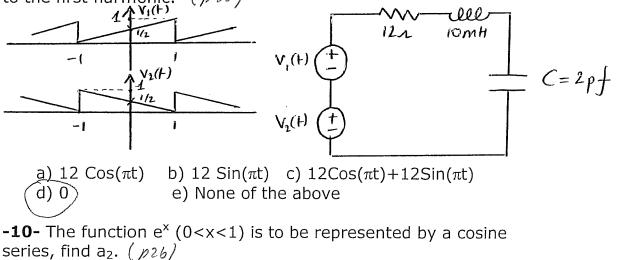
-1- Consider the two signals : $F(t) = 2+3Cos(100\pi t)+4Cos(200\pi t)+6Cos(400\pi t)$ $G(t) = Cos(100\pi t).Sin(300\pi t) - Sin(100\pi t).Cos(300\pi t)$ Sin(200πt) Find the period of each of the signals, TF and TG. (p^{25}) a) TF=1/100 TG=1/100 b) TF=1/200 TG=1/300 c) TF=1/50 TG=1/50 d) TF \rightarrow c) None of the obove TF = 1/50, G(t) = 1 d) TF=7/200 TG=1/200 -2- The signal F(t) given in (1) is an approximation of the real signal f(t) with average power equal to 50Watt. What is the %average power error in the approximation?(take $R=1\Omega$) (prs) a) 31%, b) 35% c) 65% d) 39% e) None of the above -3- Consider the trigonometric Fourier series representation of f(t) as given over the interval (-2,2): $f(t) = t+1 - 1 \le t \le 0$ $-t+1 \quad 0 \le t \le 1$ 0 elsewhere The Fourier series is also a representation of the periodic signal F(t) obtained by repeating f(t) periodically. The period of F(t) is: (p25) a) 3 b) 2 c) 1 (d) 4 e) None of the above -4- Two periodic functions of period 6 seconds each are given by: $f(t) = -t -3 < t \le 0$ $g(t) = 1 \quad 0 < t < 3$

t $0 \le t < 3$ -1 3 < t < 6Find the ratio of the amplitude of the 3^{rd} harmonic present in f(t) to that present in g(t). (p25)

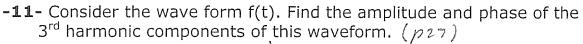
(a) $1/\pi$ b) $3/\pi$ c) $\pi/3\pi$ d) π e) None of the above

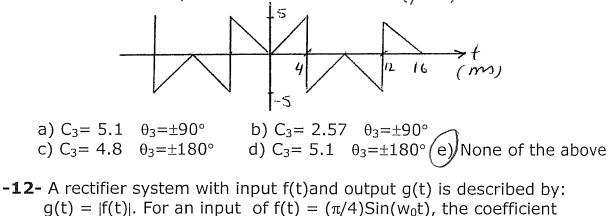


-9- Two periodic voltages V1(t) and V2(t) of the same period (T=2s) are applied to the circuit. Find the current in this circuit due to the first harmonic. (p_{26})



a)-0.684 b)-0.0827 (c)0.0848 d)0.0424 e) None of the above





of the exponential Fourier series of g(t) with n even is: (p27)

(a) $1/2(1-n^2)$ b) $1/(1-n^2)$ c) $2/(1-n^2)$ d) $1/(\pi-\pi n^2)$ e) None of the above

-13- Let the signal f(t) be a signal defined between $-\pi$ and π . F(t) is zero outside the following exponential Fourier series: (p27)

$$\sum_{n = -\infty}^{\infty} C_n \cdot e^{\left(\frac{jnt}{2}\right)}$$

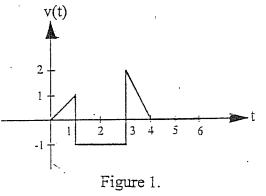
The series represents the periodic extension of f(t) with period T. Find T.

a) 8π b) 2π c) 6π d) 4π e) None of the above

-14- Consider the following signal : $F(t) = 2\cos(100\pi t) + 3\cos(300\pi t) + 6\cos(500\pi t) + 9\sin(300\pi t)$ Find the coefficients of the exponential Fourier series of F(t). (p27)

a)
$$C_1=C_{-1}=1$$
; $C_3=1.5-4.5j$; $C_{-3}=1.5+4.5j$; $C_5=C_{-5}=3$
b) $C_1=C_{-1}=-1$; $C_3=4.5-1.5j$; $C_{-3}=4.5+1.5j$; $C_5=C_{-5}=-3$
c) $C_1=C_{-1}=2$; $C_3=2.5-4.5j$; $C_{-3}=2.5+4.5j$; $C_5=C_{-5}=3$;
d) $C_1=C_{-1}=2$; $C_3=2.5-4.5j$; $C_{-3}=2.5+4.5j$; $C_5=C_{-5}=3$;
e) None of the above

- Find the rms value of v(t) in Fig. 1 over the time interval (0, 5). (p70)
 - A. $v_{rms} = 0.54$ B. $v_{rms} = 0.98$ C. $v_{rms} = 0.86$ D. $v_{rms} = 0.68$ E. None of the above



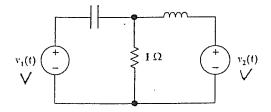
- Find the expression for the Fourier coefficients 10. C_n for the periodic function shown in Fig. 9.($\rho72$) v(t) A. $1/2\pi n$, $n \neq 0$; 1/2, n = 0B. $j/\pi n$, $n \neq 0$; 0, n = 0 \rightarrow C. j/2 π n, n \neq 0; 1/2, n = 0 D. $j/3\pi n$, $n \neq 0$; 1, n = 03 0 -3 -6 E. None of the above Figure 9. A periodic function is represented by: 11. $\mathbf{v}(t) = \sum_{\mathbf{n}=-\infty}^{+\infty} \mathbf{V}_{\mathbf{n}} \mathbf{e}^{\mathbf{j}200\pi \mathbf{n}t}$ Fig. 10 shows the plot of the magnitude of the coefficients Vn. Find the average and the fundamental frequency of v(t). $(p \gamma \lambda)$ A. 2; 1Hz B. 5; 10 Hz C. 0; 10 Hz -3 -2 -1 0 2 3 4 →D. 5; 100 Hz
 - E. None of the above

Figure 10.

14. Calculate the power dissipated in the resistor in Fig. 12 if $v_1(t) = 10cost$ and $v_2(t) = 10cos3t$. ($p \neq 3$)

A. 12.7 W - B. 50.7 W C. 60.8 W D. 70.5 W

E. None of the above



1 F

· 1 H

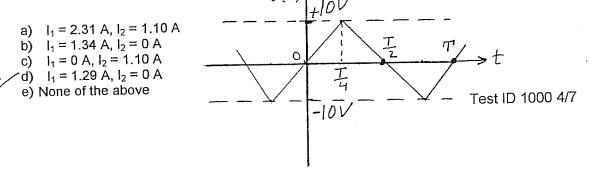
Figure 12

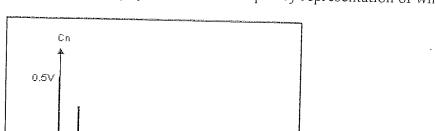
4. A series RL circuit in which R = 5 Ω and L= 20 mH has an applied voltage v=100+ 50sin ω t + 25sin 3 ω t V, with ω =500 rads/s. Find the instantaneous current. (p104)

a. i=20+4.47sin(ωt+63.4)+0.822 sin(3ωt+80.54), A b. i=20+4.47sin(ωt-63.4)+0.822 sin(3ωt-80.54), A c. i=8.96+4.47sin(ωt-63.4)+0.822 sin(3ωt-80.54), A d. i=sin(ωt-63.4)+0.822 sin(3ωt-80.54), A e. None of the above

5. Determine the power dissipated in the resistor of problem 4. p(104)

11. The figure below shows the triangular waveform of a voltage source operating at frequency f= 1kHz. Find the amplitudes of the fundamental (I_1) and the second order harmonic (I_2) current that flows through an inductor of value L= 1mH when it is supplied by this source.. (Answers are rounded to 2 digits after the decimal point) ($\rho l 0 b$)





10.0083V

7

0

1 2

34

5 6

8. The following spectrum is the frequency representation of which Fourier function: (p124)

a.
$$f(t) = \frac{4V}{\pi} \sin \omega t + \frac{4V}{3\pi} \sin 3\omega t + \frac{4V}{5\pi} \sin 5\omega t + \dots$$

b.
$$f(t) = \frac{V}{2} + \frac{4V}{\pi} \sin \omega t + \frac{4V}{3\pi} \sin 3\omega t + \frac{4V}{5\pi} \sin 5\omega t + \dots$$

c.
$$f(t) = \frac{V}{2} + \frac{4V}{\pi^2} \cos \omega t + \frac{4V}{(3\pi)^2} \cos 3\omega t + \frac{4V}{(5\pi)^2} \cos 5\omega t + \dots$$

d.
$$f(t) = \frac{V}{8} + \frac{4V}{\pi^2} \sin \omega t + \frac{4V}{(3\pi)^2} \sin 3\omega t + \frac{4V}{(5\pi)^2} \sin 5\omega t + \dots$$

> e. None of the above

n

4. A complex waveform of RMS value of 240 V has 20% 3-rd harmonic content, 5% 5-th harmonic content and 2% 7-th harmonic content. Find the RMS value of the 3-rd and 7-th harmonics respectively. (*p139*)

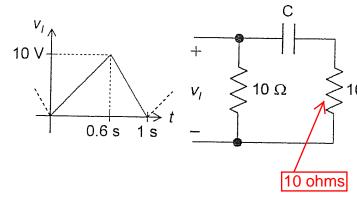
A. 11.5V, 4.6V B. 7.6V, 1.3V C. 47 V, 4.7V D. 30V, 3.2V E. None of the above 11. A voltage v(t) is applied to a 5 Ω resistor. v(t) can be written as: v(t) = 1 - $\Sigma_{n=1}^{\infty}$ (1/n²) cos (500nt)

Estimate the Power dissipated in the resistor using the first four non-zero terms of v(t). (p/4)/

- a. 2.36 W
- ---- b. 0.31W
 - c. 1.25W
 - d. 0.95W
 - e. None of the above

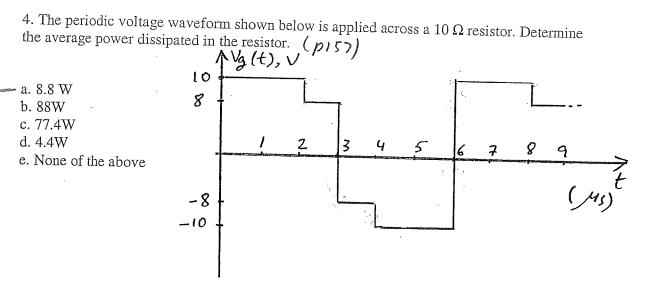
18. The periodic voltage v_l is applied to the circuit shown, the reactance of *C* at the frequency of the fundamental being much smaller than 10 Ω . Determine the power dissipated in the circuit. (*p*144)

- A. 13.33 W
- B. 8.33 W
- C. 6.67 W
- = D. 4.17 W
 - E. None of the above



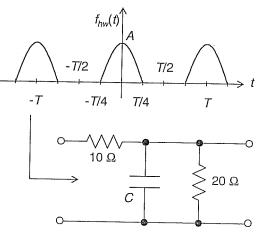
20. The current through a 1 μ F capacitor is 2 cos² 100 π t mA, where *t* is in s. Determine the period of the voltage across the capacitor.

- A. 25 ms
- B. 50 ms
- C. 100 ms
- D. 200 ms
- E. None of the above



9. A half-wave rectified waveform f_{hw}(t) of frequency 50 Hz and having A = 10 V is applied to the circuit shown, where the reactance of C is negligible at 50 Hz.
Determine the total power dissipated in the circuit. (p158)

- A. 0.83 W
- B. 1.49 W
- ---- C. 1.82 W
 - D. 2.5 W
 - E. None of the above



8%

- 4. A voltage having the waveform of the figure of Problem 9 below, with A = 8 V and T
 - = 1 s is applied to a coil having a resistance of 4 Ω and an extremely large inductance. Determine the average power dissipated in the coil. (*p1 70*)
 - A. 1.56 W



C. 3.28 W

- D. 4 W
- E. None of the above

Problem 9

Derive the trigonometric form of the FSE of the waveform

shown

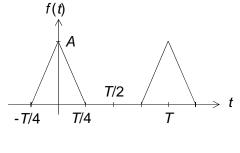
Solution: The function is even, and $a_0 = C_0 =$

$$\frac{1}{T} \times A \times \frac{T}{4} = \frac{A}{4}; \ a_n = \frac{4A}{T} \int_0^{T/4} \left(-\frac{4}{T} t + 1 \right) \cos n\omega_0 t dt$$

$$= \frac{4A}{T} \left[-\frac{4}{T} \frac{1}{n^2 \omega_0^2} \cos n\omega_0 t - \frac{4}{T} \frac{t}{n\omega_0} \sin n\omega_0 t - \frac{1}{n\omega_0} \sin n\omega_0 t \right]_0^{T/4}$$

$$= \frac{16A}{T^2 n^2 \omega_0^2} \left[1 - \cos \frac{n\pi}{2} \right] = \frac{4A}{\pi^2 n^2} \left(1 - \cos \frac{n\pi}{2} \right). \text{ Hence,}$$

$$f(t) = \frac{A}{4} + \frac{4A}{\pi^2} \left(\cos \omega_0 t + \frac{1}{2} \cos 2\omega_0 t + \frac{1}{9} \cos 3\omega_0 t + \frac{1}{25} \cos 5\omega_0 t + ... \right).$$

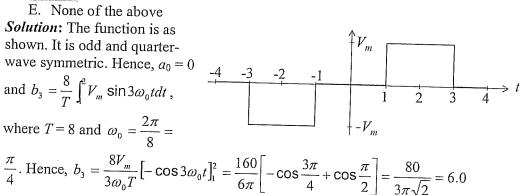


6. Consider a periodic function f(t), described by the following sequence during one period of time:

$\mathbf{f}(\mathbf{t}) = 0$	for	$-4 \le t < -3$
f(t) = -Vm	for	$-3 \le t < -1$
$\mathbf{f}(\mathbf{t})=0$	for	$-1 \le t < +1$
f(t) = +Vm	for	$+1 \le t < +3$
$\mathbf{f}(\mathbf{t})=0$	for	$+3 \le t \le +4$

where Vm = 20. Find the amplitude "A" of the third order harmonic in the Fourrier series, expressed by $A\cos(3\omega t - \Theta)$. (p/7b)

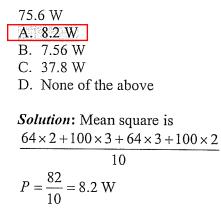
- A. A=3
- B. A=4
- C. A=5
- D. A=6



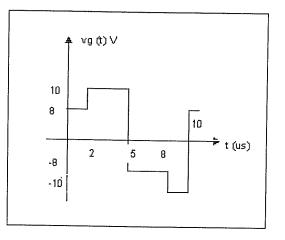
11. A voltage $5\sin\omega_0 t$ V applied to a given resistor dissipates 5 W. What is the power dissipated by a voltage $5|\sin\omega_0 t|$ V applied to the same resistor? (p179) A. 5 W

- B. $5\sqrt{2}$ W
- C. $5/\sqrt{2}$ W
- D. 10 W
- E. None on the above

Solution: The two waveforms have the same rms value and would therefore dissipate the same power in a given resistor.



= 82.



1/1

- in the circuit shown, each source is 15cos10t V. The power dissipated in *R* is 50 W. If the frequency of one of the sources is doubled, the power dissipated in *R* is:
 - A. 100 W
 - B. 50W
 - <mark>C. 25 W</mark>
 - D. 12.5 W
 - E. None of the above.

Solution: The current due to each source is $\frac{1}{2}\left(\frac{15}{1.5}\right)\cos 10t = 5\cos 10t$ A. The power is

 $\left(\frac{5}{\sqrt{2}}\right)^2 = 12.5 \,\text{W}.$ the power dissipated due to both

sources is 25 W.

2. $f_2(t)$ is the function $f_1(t)$ lowered by 1 unit, as shown. If F_{1rms} and F_{2rms} are the rms values of $f_1(t)$ and $f_2(t)$, respectively, then:

A.
$$F_{1rms} = F_{2rms}$$

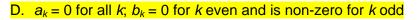
$$\mathsf{B.} \quad F_{1\mathrm{rms}} > F_{2\mathrm{rms}}$$

C.
$$F_{1rms} < F_{2rms}$$

Solution: The AC components of $f_1(t)$ and $f_2(t)$ are

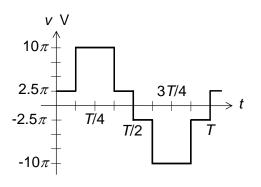
the same. The DC component of $f_1(t)$ is larger than that of $f_2(t)$. Hence, $F_{1rms} > F_{2rms}$.

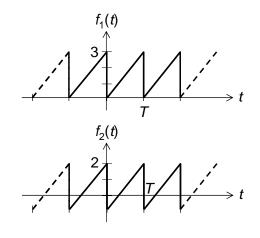
- 5. The Fourier coefficients a_k and b_k for the periodic function shown are:
 - A. $a_k = 0$ for all k; $b_k = 0$ for k odd and is non-zero for k even
 - B. $b_k = 0$ for all k; $a_k = 0$ for k even and is non-zero for k odd
 - C. $b_k = 0$ for all k; $a_k = 0$ for k = 0, $a_k = 0$ for k odd and is non-zero for k even

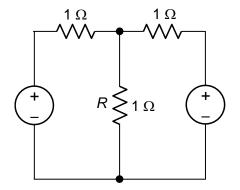


E. None of the above.

Solution: The function is odd, half-wave symmetric. Its average is zero; it contains no cosine terms, only odd sine terms.







- 11. The current through an inductor of 1 H is given by the periodic triangular wave. The amplitude of the fundamental component of the voltage across the inductor is:
 - A. 4*I*_p
 - B. 8*I*_p
 - C. 16*I*_p
 - D. 32*1*_p
 - E. None of the above.

Solution: $v = L \frac{di}{dt} = 1 \times \frac{2I_p}{1/4\pi} = 8\pi I_p$, which is the amplitude of the square waveform

representing v. the amplitude of the fundamental is $\frac{4 \times 8\pi l_p = 32 l_p}{\pi}$.

15. The voltage and current at the terminals of a circuit are:

 $v = 15 + 400 \cos 500 t + 100 \sin 1500 t$ V

 $i = 2 + 5\sin(500t + 60^{\circ}) + 3\cos(1500t - 15^{\circ})$ A

3% a) Calculate the average power delivered to the circuit.

$$P = V_{dc}I_{dc} + \sum_{n=1}^{3} \frac{V_m I_m}{2} \cos(\theta_{vn} - \theta_{in}) = 15 \times 2 + \frac{1}{2} \times 400 \times 5\cos(30^\circ) + \frac{1}{2} \times 100 \times 3 \times \cos(-75^\circ)$$

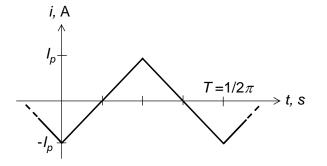
= 934.85 W

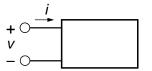
3% b) Calculate the rms value of v.

$$V_{rms} = \sqrt{(15)^2 + \frac{(400)^2}{2} + \frac{(100)^2}{2}} = 291.93$$
 V

2% c) Calculate the rms value of *i*.

$$I_{rms} = \sqrt{(2)^2 + \frac{(5)^2}{2} + \frac{(3)^2}{2}} = 4.58$$
 A





12. For $n = 1, 2, 3, \dots$, the function shown has:

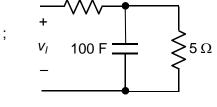
- A. a_n and b_n nonzero for all n
- B. a_n and b_n are zero for even n
- C. a_n and b_n are zero for odd n
- D. $a_n = 0$ for all n
- E. $b_n = 0$ for all n

 $P_{dc} = \frac{4V_m^2}{10\pi^2} = 0.04053V_m^2;$

Solution: When the dc value is removed, the ac

component has half-wave symmetry but is neither even nor odd. Hence, a_n and b_n are zero for even n.

13. Determine the total power dissipated if v_l is a full-wave rectified waveform given by: $v_l = 6|\sin(500t)|$ V. **Solution:** $\frac{1}{\omega C} = \frac{1}{500 \times 100} = 2 \times 10^{-5} << 5$ ohms; $V_{dc} = \frac{2V_m}{\pi}$;



5Ω

f(t)

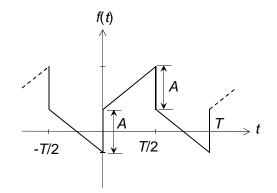
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$$\frac{V_m^2}{2} = \frac{4V_m^2}{\pi^2} + V_{ac}^2; \ V_{ac}^2 = V_m^2 \left(\frac{1}{2} - \frac{4}{\pi^2}\right) = 0.09472V_m^2; \ P_{ac} = \frac{0.09472V_m^2}{5} \ 0.01894V_m^2$$
$$P = 0.05947V_m^2.$$

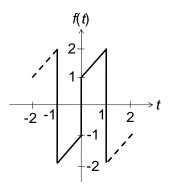
14. A period of a periodic function f(t) is given by: $K(4 + 2\sin t)$, $0 < t < 2\pi$. Determine the rms value of f(t), if K = 0.5.

Solution: The square of f(t) is $K^2(16 + 16 \sin t + 4 \sin^2 t) =$

 $\mathcal{K}^{2}(16+2+16\sin t-2\cos 2t). \text{ The area under the square is} \xrightarrow{\pi} 2\pi \xrightarrow{2\pi} t$ $\int_{0}^{2\pi} \mathcal{K}^{2}(16+2+16\sin t-2\cos 2t)dt = 36\pi \mathcal{K}^{2}; \text{ the mean square is } \frac{36\pi \mathcal{K}^{2}}{2\pi} = 18\mathcal{K}^{2} \text{ and the rms}$ value is $3\sqrt{2}\mathcal{K}$.

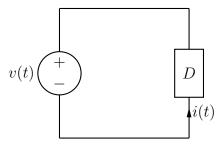


16. Derive the trigonometric Fourier expansion of the given periodic function f(t).



Solution: Since
$$f(t)$$
 is odd, $a_0 = 0 = a_n$; $T = 2$, $\omega_0 = 2\pi/T = \pi$; $f(t) = t + 1$;
 $b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin(n\omega_0 t) dt = 2 \operatorname{Im} \left[\int_0^{T/2} f(t) e^{jn\omega_0 t} dt \right] = 2 \operatorname{Im} \left[\frac{t e^{jn\omega_0 t}}{jn\omega_0} - \frac{e^{jn\omega_0 t}}{(jn\omega_0)^2} + \frac{e^{jn\omega_0 t}}{jn\omega_0} \right]_0^1 = 2 \operatorname{Im} \left[\frac{e^{jn\pi}}{jn\pi} + \frac{e^{jn\pi}}{(n\pi)^2} + \frac{e^{jn\pi}}{jn\pi} - 0 + \frac{1}{n^2\pi^2} - \frac{1}{jn\pi} \right] = 2 \left[-\frac{2\cos n\pi}{n\pi} + \frac{1}{n\pi} \right] = \frac{2}{\pi} \left[\frac{1}{n} (1 - 2\cos n\pi) \right] = \frac{2}{\pi} \left[\frac{1}{n} (1 + 2(-1)^{n+1}) \right] f(t) = \frac{2}{\pi} \left[3\sin \pi t - \frac{\sin 2\pi t}{2} + \sin 3\pi t - \frac{\sin 4\pi t}{4} + \dots \right].$

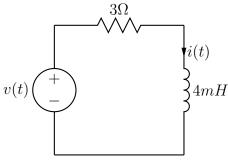
The device D in the following circuit is powered by a voltage $v(t) = 2 + 2\cos(1000t) + \cos(2000t)(V)$. The current across is given by $i(t) = 1 + \sin(1000t) + 0.5\sin(2000t)(A)$ find the average power associated with D.



- A) 2W
- →B) -2W
 - C) 4W
 - D) -4W
 - E) None of the above

Problem 19

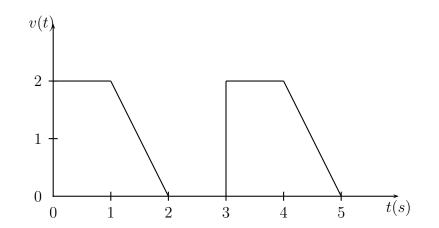
The following circuit is powered by a periodic voltage source that has the following Fourier expansion: $v(t) = 21 + 20\cos(1000t) + 10\cos(2000t)$. Find the RMS value of the current i(t).



- A) 6.68A
- B) 8.14A
- C) 7.3A
- →D) 7.6A
 - E) None of the above

For the function v(t) given below, find the value of a_5 . You may use the following:

$$\int t \cos(\alpha t) = \frac{\cos(\alpha t)}{\alpha^2} + \frac{t \sin(\alpha t)}{\alpha}$$

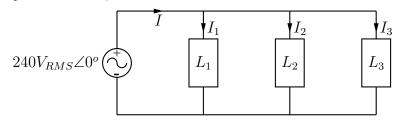


→A) 0

•

- B) -0.0162
- C) 1.27
- D) -16.7
- E) None of the above

The following given is used in the next 5 problems. 3 electrical elements are powered by a $240V_{RMS}$, 60Hz source:



The following is given for the three elements:

- L1: 240W, PF=0.6 Lag
- L2: 200VARS, PF=0.5 Lag
- L3: 100VA, PF=0 Lead

Problem 11

Find the total apparent power.

- A) 725.67VA
- B) 626.33VA
- →C) 550.2VA
 - D) 888.8VA
 - E) None of the above

Problem 12

Find the total power factor.

- A) 0.567 Lag
- \rightarrow B) 0.646 Lag
 - C) 0.808 Lag
 - D) 0.747 Lag
 - E) None of the above

Find the magnitude of the total current I.

- A) 5.21A
- B) 3.02A
- → C) 2.292A
 - D) 7.40A
 - E) None of the above

Problem 14

Find the capacitor that needs to be placed in parallel with the loads to adjust the power factor to 0.9 Lag.

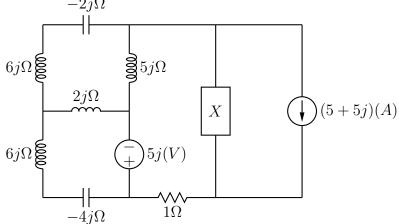
- A) $62.7\mu F$
- B) $147\mu F$
- C) $49\mu F$
- D) $11.4 \mu F$
- \rightarrow E) None of the above C=2.74 mF

Problem 15

Find the magnitude of I again after the power factor is adjusted as in the previous problem.

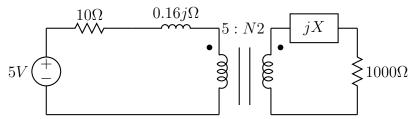
- →A) 1.64A
 - B) 3.29A
 - C) 0.91A
 - D) 2.13A
 - E) None of the above

Find X such that the maximum power transfer constraint is satisfied. $-2j\Omega$



- \rightarrow A) $1 2.5j\Omega$
 - B) $1 + 2.5j\Omega$
 - C) $2 2.5j\Omega$
 - D) $2 + 2.5j\Omega$
 - E) None of the above

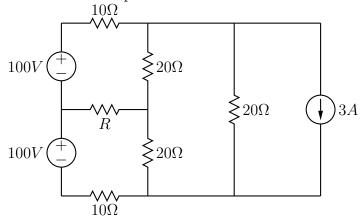
Find N_2 and X such that maximum power is delivered to the 1000 Ω resistor.



A)
$$N_2 = 50, X = -16$$

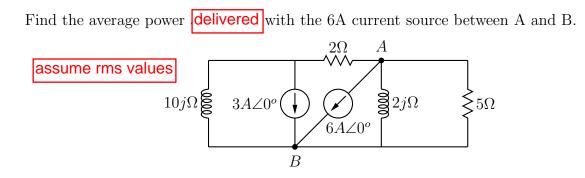
B) $N_2 = 10, X = -16$
C) $N_2 = 10, X = -0.64$
D) $N_2 = 2, X = -0.64$
E) None of the above

Find R that satisfies the maximum power transfer constraint.



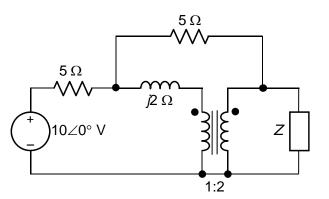
A) 14Ω

- \rightarrow B) 15 Ω
 - C) 16.33Ω
 - D) 17.46Ω
 - E) None of the above



- A) -23.86W
- →B) 23.86W
 - C) 28.71W
 - D) -28.71W
 - E) None of the above

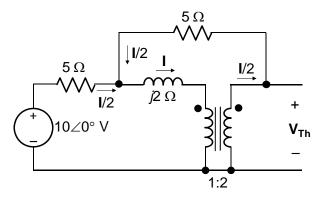
18. Determine Z so that maximum power is transferred to it and calculate this power given that the source voltage is10 V peak value.

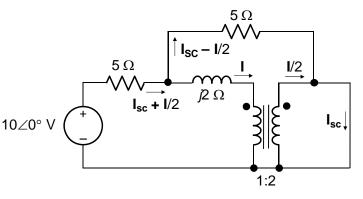


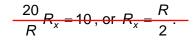
Solution: We will determine TEC as seen by Z. On open circuit, the currents are as shown. From KVL: $10 \angle 0^\circ - 5I/2 + 5I/2 =$ V_{Th} . In This particular problem, the voltages across the 5 Ω resistors cancel out. Hence, $V_{Th} = 10 \angle 0^\circ$ V peak value

When Z is replaced by a short circuit, the currents are as shown. From KVL: $10 \ge 0^\circ - 5(I_{sc} + I/2) - 5(I_{sc} - I/2) = 0$. Again, the terms involving I cancel out. Hence, I_{sc} = $1 \ge 0^\circ$ A, and $Z_{Th} = \frac{10 \ge 0^\circ}{1 \ge 0^\circ} = 10 \Omega$. It follows that for maximum power transfer, Z = 10 Ω . The power dissipated in the

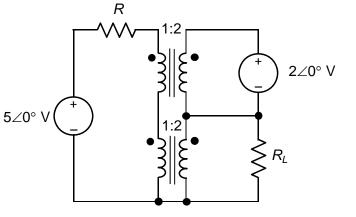
load is
$$\left(\frac{V_{Th}}{\sqrt{2}}\right)^2 \frac{1}{4 \times 10} = 1.25 \text{ W}.$$







- 7. Determine the maximum power that can be delivered to R_L , assuming R= 0.5 Ω .
- **Solution:** The primary voltage of the upper transformer is always 1 V. On

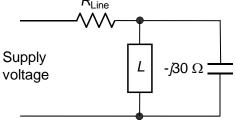




open circuit, the source current is zero, the primary voltage is 5 - 1 = 4 V, and $V_{Th} = 8$ V. On short circuit, the primary voltage of the lower transformer is zero, the source current is (5 - 1)/R and the short circuit current is 2/R. This gives, $R_{Th} = 4R$. The maximum power delivered is $(8)^2/(4 \times 4R) = 4/R$.

8. Given that the load *L* consumes 1200 W at 0.8 p.f. lagging and the magnitude of the voltage across *L* is 300 V rms. Determine the power dissipated in the resistance R_{line} , if $R_{\text{line}} = 0.5 \Omega$.

Solution: The reactive power absorbed by the load



is $\frac{1200}{0.8} \times 0.6 = 900$ VAR. The reactive power absorbed by the capacitor is $\frac{V^2}{-30} = -3000$

VAR. The total complex power is 1200 + j(900 - 3000) = 1200 - j2100 VA. The magnitude of

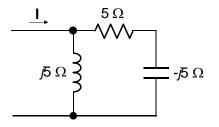
the line current $\frac{\sqrt{(1200)^2 + (2100)^2}}{300} = \sqrt{65}$ A. The power dissipated in R_{line} is $65R_{\text{line}}$.

4. Determine the reactive power absorbed in the circuit,

given that $I = 1 \angle 0^{\circ}$ A rms.

Solution: The equivalent series impedance is

$$\frac{j5(5-j5)}{j5+5-j5} = 5+j5$$
. The reactive power is $5|I|^2$ VAR. As a

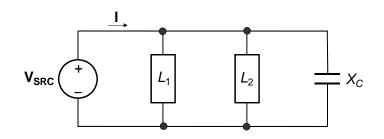


check, the current in the capacitive branch is $\frac{j5}{j5+5-j5}$ I = *j*I; the reactive power absorbed

by the capacitor is $-5|j|^2 = -5|j|^2$ VAR. The current in the inductive branch is $\frac{5-j5}{j5+5-j5}I = (1)$

− *j*)I = $\sqrt{2} \angle -45^{\circ}$ I; the reactive power absorbed by the inductor is $5|\sqrt{2}|^2 = 10|/|^2$ VAR. The total reactive power absorbed is $10|/|^2 - 5|/|^2 = 5|/|^2$ VAR.

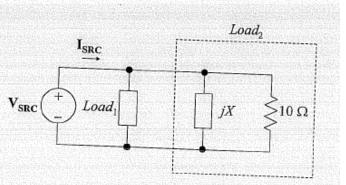
5. In the circuit shown, L_1 consumes 160 W at 0.8 p.f. lagging and L_2 consumes 320 VAR at 0.6 p.f. lagging. Determine I when X_C is chosen for unity power factor, assuming $V_{SRC} = 200 \angle 0^\circ$ V rms.



Solution: At unity p.f. the total reactance seen by the source is zero and the source applies only real power. The real power consumed by L_2 is $\frac{320}{0.8} \times 0.6 = 240$ W. The total real power supplied by the load is 160 + 240 = 400 W. The current is $\frac{400}{V_{SRC} \angle 0^{\circ}} = \frac{400}{V_{SRC}} \angle 0^{\circ}$ A

8%

 The complex powers absorbed by L₁ and L₂ are 1 + j0.2 kVA and 1 - j0.2 kVA. Determine Isrc, assuming that the phase angle of V_{SRC} is zero. Note that X need not be given.



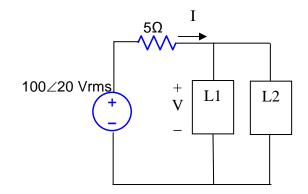
A. 20∠90° A	B.	10∠90°A
C. 10∠0° A	→D.	20∠0° A
T >-		

E. None of the above

Solution: The complex power delivered by the source is 2 kVA. The real power absorbed

by
$$L_2$$
 is in the 10 Ω resistor. If $\mathbf{V}_{SRC} = V_m \angle 0$ V, then $\frac{|V_m|^2}{10} = 1000$, or $V_m = 100$ V. It

follows that $I_m = \frac{2000}{100} = 20$ A, and $\mathbf{I}_{SCR} = 20 \angle 0$ A.



It is given that the complex power of L1 is 5+j10 VA. It is also given that L2 absorbs 20W at lagging power factor of 0.8. What is the phase difference between I and V as shown in figure?

A) 45.00°
B) 39.81°
C) 63.33°
D) 60.00°
E) None of the above.

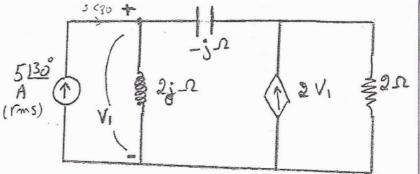
Problem 15

What is the impedance of a load if it absorbs 20KVAR at lagging power factor of 0.6 when a current of magnitude 50 A rms flows through it?

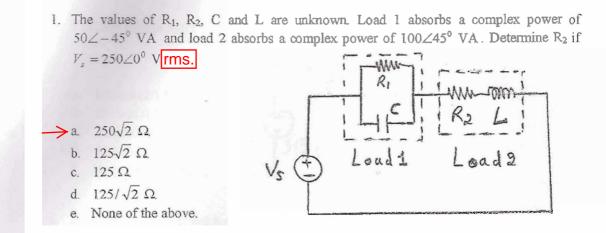
A) 6 + 8j ohms
 B) 3 + 4j ohms
 C) 4+ j3 ohms
 D) 8+ j6 ohms
 E) None of the above.

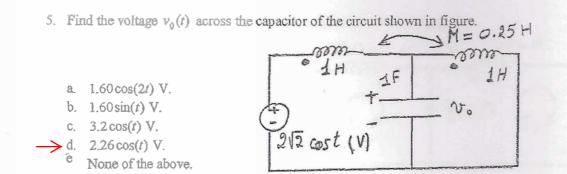
4. How much complex power is delivered by the 5∠30° A (rms) current source to the circuit shown in figure.

- a. 7.5∠137.48° VA.
- b. 0 VA.
- c. 100 VA.
- →d. 15.35∠137.48° VA.
 - e. None of the above.



2/4





- 6. Two impedances $Z_1 = 9.8 \angle -78^{\circ} \Omega$ and $Z_2 = 18.5 \angle 21.8^{\circ} \Omega$ are connected in parallel and the combination in series with an impedance $Z_3 = 5 \angle 53^{\circ} \Omega$. If this circuit is connected across a 100-V source (rms), how much average power will be supplied by the source.
- → a, 980.8 W.
 - Ъ. 490 W.
 - c. 1960 W.
 - d. 1391.6 W.
 - e. None of the above.

An impedance Z1= (4+j4) Ω is connected in parallel with an impedance Z2= (12+j6) Ω. If the input reactive power is 1000 VAR (lagging), what is the total active (average) power ?

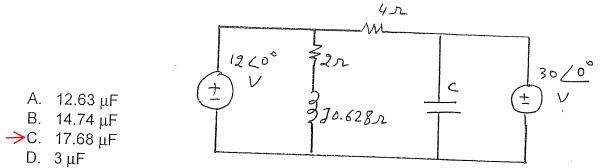
- →A. 1210 W
 - B. 3025 W
 - C. 826.39 W
 - D. 1150 W
 - E. None of the above

- 3. The conjugate of the complex power delivered by a current source is 200 j200 VA. If the source current is $\frac{10}{\sqrt{2}} \angle 45^{\circ}$ A peak, determine the rms voltage across the source.
- A. 40 V rms
- B. *j*40 V rms
- C. 80 V rms
- D. --*j*40 V rms

 \rightarrow E. None of the above

j40sqrt(2) rms

11. Determine the value of C in the circuit shown if C takes 5 VAR. The operating frequency is 50 Hz.

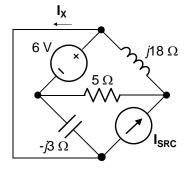


E. None of the above

- 2. In the circuit shown, the capacitance absorbs -200 VAR. Determine the average power dissipated in *R* if $R = 5 \Omega$.
 - A. 57.1 W
 - <mark>B. 80 W</mark>
 - C. 44.4 W
 - D. 66.7 W
 - E. 50 W

Solution: $Q = -BV_{\text{rms}}^2$, where V_{rms} is the rms voltage across R and C, and B = -1/X = 1/2 S. Substituting, $-200 = -\frac{1}{2}V_{\text{rms}}^2$, and $V_{\text{rms}} = 20$ V. It follows that $P_R = \frac{V_{\text{rms}}^2}{R} = \frac{400}{5} = 80$ W.

- **3.** Determine I_X assuming $I_{SRC} = j A$.
 - A. *j*6 A
 - В. -*ј*ЗА
 - <mark>С.</mark> јЗА
 - D. *-j*6 A
 - E. *j*4 A



-j2 Ω

8Ω

*j*20 Ω

50∠0° V

 \sim

Solution: The voltage across the $-j3 \Omega$ capacitor is 6 V and the current through this capacitor, directed upwards is j2 A. It follows that $I_x = I_{SRC} + j2 = j3$ A.

- 5. Two coils are tightly coupled to a high-permeability core, so that the leakage flux is negligibly small. If coil 1 has 100 turns and an inductance of 10 mH, and the mutual inductance is 12.5 mH, determine the number of turns of coil 2.
 - <mark>A. 125</mark>
 - B. 250
 - C. 150
 - D. 175
 - E. 200

Solution: From the definitions of self and mutual inductance, with negligible leakage flux,

$$L_1 = \frac{N_1 \phi_{21}}{i_1}$$
 and $M = \frac{N_2 \phi_{21}}{i_1}$. It follows that $N_2 = \frac{M}{L_1} N_1 = 10M = 125$.

- 6. Determine the inductance of coil 2 of the preceding problem.
 - A. 22.5 mH
 - B. 30.63 mH
 - C. 15.63 mH
 - D. 40 mH
 - E. 50.63 mH

Solution: Since the coils are tightly coupled to the core, k = 1, so that $M^2 = L_1 L_2$, or

$$L_2 = \frac{M^2}{L_1} = 0.1M^2$$
 mH. It also follows from the solution of the preceding problem that

$$N_1 = \frac{M}{L_2} N_2$$
. Dividing, $L_2 = L_1 \left(\frac{N_2}{N_1}\right)^2 = 0.1 M^2 = 0.1 \times (12.5)^2 = 15.625 \text{ mH}.$

- **7.** A D'Arsonval movement has a resistance of $R \Omega$ and a full-scale deflection of 100 μ A. Determine the shunt resistance that will result in a full-scale deflection of 150 μ A, assuming $R = 50 \Omega$.
 - A. 150 Ω
 - Β. 200 Ω
 - C. 300Ω
 - D. 100 Ω
 - E. 250 Ω

Solution: At full-scale deflection, the voltage drop across the movement and shunt is ($R = \Omega$)×(100 µA) = 100R µV. The shunt has to pass 50 µA, so its resistance is $R_{\text{shunt}} = 100R/50 = 2R = 100 \Omega$.

- 8. When a 9950 Ω resistance is connected in series with a D'Arsonval movement of unknown resistance and full-scale deflection current, a voltage of 1 V across the series combination gives a certain full-scale deflection. If an additional 10,000 Ω is connected in series with the combination, 2 V are required for full-scale deflection. Determine the resistance of the D'Arsonval movement.
 - A. 150 Ω
 - B. 100 Ω
 - **C**. 75 Ω
 - D. 125 Ω
 - <mark>Ε. 50 Ω</mark>

Solution: Let the resistance of the movement be R_m , its FSD current be I_{FSD} , and the FSD voltage with series resistance be V_{FSD} . Then $I_{FSD}(R + R_m) = V_{FSD}$, and $I_{FSD}(10,000 + R + R_m) = 2V_{FSD}$. It follows that $R + R_m = 10,000$, or $R_m = 10,000 - R = 50 \Omega$.

- **9.** Determine L_{eq} if L = 1 H.
 - A. 6 H
 - <mark>B. 4 H</mark>
 - C. 8 H
 - D. 7 H
 - E. 5 H

 $\begin{array}{c}
1H\\
1H\\
1H\\
2H\\
3H \bullet L
\end{array}$

>10 Ω

 V_{SRC}

V_{SRC}

 V_{SRC}

Iχ

*i*10 Ω

10Ω+

*j*10 Ω

1:2

2V_{SRC}

V_{Th}

 Z_L

+

 V_{Th}

Solution: Consider that a voltage V is applied, causing a current I to flow. $V = j\omega I[(2 - 1 + 1) + (3 - 1 - 1) + (L + 1 - 1)]; L_{eq} = 3 + L = 4$ H.

10. Determine V_{Th} , assuming $V_{SRC} = 1 \angle 0^{\circ} V$

- <mark>A. -1∠0°</mark> V
- B. 1∠0° V
- C. -2∠0° V
- D. $2\angle 0^\circ V$
- E. 4∠0° V

Solution: On open circuit, no current flows. The primary voltage

is V_{SRC} as shown, and V_{Th} = - V_{SRC} = -1 $\angle 0^\circ$ V

- **11.** Determine Z_L for maximum average power delivered to it if $R = 5 \Omega$ and $I_X = k \angle -45^\circ$ where $k = \sqrt{2}$ A rms.
 - A. 10 + *j*10 Ω
 - B. 5 + *j*5 Ω
 - <mark>C. 5 *j*5 Ω</mark>
 - D. 10 *j*10 Ω
 - E. 15 *j*15 Ω

Solution: Z_{Th} is $(R + j5) \Omega$. Hence, Z_L for maximum power transfer is $(R - j5) = (5 - j5) \Omega$.

100∠0° V

rms

12. Determine the maximum average power delivered to Z_L in Problem 11, assuming that R

= 5 Ω and I_X is as in Problem 11.

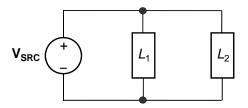
- A. 90 W
- B. 200 W
- C. 320 W
- D. 180 W
- E. 245 W

Solution: V_{Th} as seen by Z_L is determined from superposition as $\frac{j10}{j10+j10} \times 100 \angle 0^\circ$ +

$$(5+j10 || j10)\mathbf{I}_{\mathbf{X}} = 50\angle 0^{\circ} + 5(1+j)\mathbf{I}_{\mathbf{X}} = 50\angle 0^{\circ} + (5\sqrt{2}\angle 45^{\circ}) \times k\angle -45^{\circ} = 50 + 5\sqrt{2}k;$$

$$P_{L\max} = \frac{\left(50 + 5\sqrt{2}k\right)^2}{4R_{Th}} = \frac{\left(50 + 5\sqrt{2}k\right)^2}{20} = \frac{\left(50 + 5\sqrt{2} \times \sqrt{2}\right)^2}{20} = \frac{\left(60\right)^2}{20} = 180 \text{ W}.$$

13. Load L_1 absorbs 15 kVA at 0.6 p.f. lagging, whereas Load L_2 absorbs 4.8 kW at 0.8 p.f. leading. If **V**_{SRC} = 200∠0° V rms at *f* = 50 Hz, determine the capacitor that must be connected in parallel with L_1 and L_2 to have maximum magnitude of current through the source.



A. 0.67 mF

- B. 0.55 mF
- C. 0.34 mF
- D. 0.46 mF
- E. 1.24 mF

Solution: The reactive power absorbed by L_1 is 15×0.8 kVAR = 12 kVAR, whereas the reactive power absorbed by L_2 is $-\frac{4.8}{0.8} \times 0.6 = -3.6$ kVAR. For maximum magnitude of source current, the p.f. should be unity. The capacitor must add a reactive power of -(12 – 3.6) = -8400 VAR. hence, -8400 = $-\omega C \times |\mathbf{V}_{SRC}|^2$, or $C = \frac{8400}{100\pi} |\mathbf{V}_{SRC}|^2 = \frac{84}{\pi} |\mathbf{V}_{SRC}|^2 = \frac{84}{\pi} |\mathbf{V}_{SRC}|^2$

$$\frac{84}{\pi (200)^2} \equiv 0.67$$
 mF.

14. A periodic current is shown, where over a

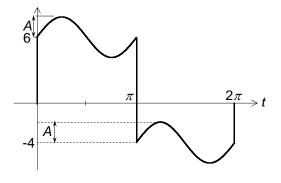
period,

$$i = 6 + A \sin 2t$$
 $0 \le t \le \pi$

$$i = -4 + A \sin 2(t - \pi)$$
 $\pi 0 \le t \le 2\pi$

Determine the rms value of *i* if A = 1 A.

- A. 5.83 A
- <mark>B. 5.15 A</mark>
- C. 6.20 A
- D. 5.29 A
- E. 5.52 A



Solution: The waveform consists of three components: i) a dc component of 1 A, ii) a square wave of 5 V amplitude, and iii) a sinusoidal wave of amplitude *A*. It follows that the rms value is $I = \sqrt{1^2 + 5^2 + A^2/2} = \sqrt{26 + A^2/2} = \sqrt{26.5} = 5.15 \text{ A}.$

- **15.** The current waveform of the preceding problem is applied to a 2 Ω resistor in parallel with a very large capacitor. Determine the voltage across the parallel combination.
 - A. 2.5 V
 - B. 2 V
 - C. 3 V
 - D. 4 V
 - E. 3.5 V

Solution: The ac voltage will be negligibly small. The dc voltage is the dc component of current multiplied by *R*, or $V = 1 \times R = 2$ V.

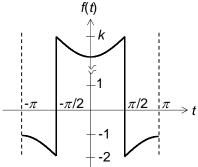
16. The period of a periodic function f(t) is defined as:

$$f(t) = \cos(t + \pi) - 2, \qquad -\pi < t < -\pi/2$$

$$f(t) = -\cos(t) + k, \qquad -\pi/2 < t < +\pi/2$$

$$f(t) = \cos(t - \pi) - 2, \qquad \pi/2 < t < \pi$$

Derive the trigonometric Fourier series expansion of f(t), assuming k = 3.



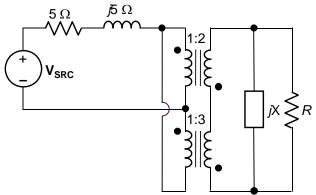
Solution:
$$a_{0} = \frac{1}{\pi} \left[\int_{0}^{\pi/2} (-\cos t + k) dt + \int_{\pi/2}^{\pi} (\cos(t - \pi) - 2) dt \right] = \frac{1}{\pi} \left[k \int_{0}^{\pi/2} dt - 2 \int_{\pi/2}^{\pi} dt - \int_{0}^{\pi/2} \cos t dt - \int_{\pi/2}^{\pi} \cos t dt \right] = \frac{1}{\pi} \left[k \int_{0}^{\pi/2} dt - 2 \int_{\pi/2}^{\pi} dt \right] = \frac{1}{\pi} \left[\frac{k\pi}{2} - \pi \right] = \frac{k}{2} - 1$$

$$a_{n} = \frac{2}{\pi} \left[\int_{0}^{\pi/2} (-\cos t + k) \cos nt dt + \int_{\pi/2}^{\pi} (\cos(t - \pi) - 2) \cos nt dt \right] = \frac{2}{\pi} \left[-\int_{0}^{\pi} \cos t \cos nt dt + \int_{0}^{\pi/2} k \cos nt dt - \int_{\pi/2}^{\pi} 2 \cos nt dt \right] = \frac{2}{\pi} \left[-\frac{1}{2} \int_{0}^{\pi} \cos(n - 1) t dt - \frac{1}{2} \int_{0}^{\pi} \cos(n + 1) t dt + k \int_{0}^{\pi/2} \cos nt dt - 2 \int_{\pi/2}^{\pi} \cos nt dt \right] = -\frac{1}{\pi} \left[\frac{\sin(n - 1)t}{n - 1} + \frac{\sin(n + 1)t}{n + 1} \right]_{0}^{\pi} + \frac{2k}{n\pi} [\sin nt]_{0}^{\pi/2} - \frac{4}{n\pi} [\sin nt]_{\pi/2}^{\pi} = 0 - 0 + \frac{2k}{n\pi} \sin \frac{n\pi}{2} - 0 - 0 + \frac{4}{n\pi} \sin \frac{n\pi}{2} = \frac{2(k + 2)}{n\pi} \sin \frac{n\pi}{2} \cdot a_{n}$$
 is zero for even values, and the

odd harmonics alternate in sign. Thus,

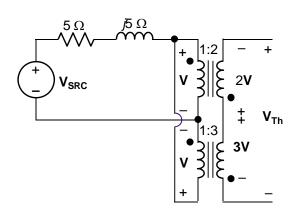
$$f(t) = \left(\frac{k}{2} - 1\right)\frac{1}{2} + \frac{2(k+2)}{\pi}\left(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \frac{1}{7}\cos 7t + \dots\right)$$
$$= \frac{1}{2} + \frac{10}{\pi}\left(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \frac{1}{7}\cos 7t + \dots\right).$$

19. Determine X and R for maximum power transfer to R and calculate this power. Assume $V_{SRC} = 4 \angle 0^\circ$ V rms.



Solution: On open circuit, $V_{TH} = V = V_{SRC}$. On short circuit, V = 0 and $I_N = I = \frac{V_{SRC}}{5(1 + j)}$. $Y_N =$

 $\frac{1}{5(1+j)} = 0.1(1-j)$ S. For maximum power transfer, $G_L = 0.1$ S, or $R = 10 \Omega$, and $B_L = 0.1$ S, or $X = -10 \Omega$.

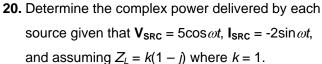


Under conditions of maximum power transfer, the current in *R* is $0.5|\mathbf{I}_N| = \frac{0.5 |\mathbf{V}_{SRC}|}{10}$ and the power transferred is $\frac{0.25 |\mathbf{V}_{SRC}|^2}{10} = \frac{|\mathbf{V}_{SRC}|^2}{40} = \frac{16}{40} = 0.4$ W.

 $\frac{1}{10} = \frac{1}{10}$ wer delivered by each $\cos \omega t$, $I_{SRC} = -2\sin \omega t$,

5Ω

 V_{SRC}



Solution: The currents and voltages are as shown. Equating mmfs: $100 \times 2 \angle 90^{\circ} + 200I_{L} = 0$, or $j^{2} = -2I_{L}$, and $I_{L} = -j A$, $I_{1} = I_{L} - j^{2} = -j^{3} A$. $V_{L} = Z_{L}I_{L} = -jk(1 - j) = -k(1 + j) \vee V_{2} = V_{L} - 5 = -(k + 5) - jk$. $V_{1} = V_{2}/2 = -(k + 5)/2 - jk/2 \vee V_{1} = 5 - V_{1} = 5$

$$(15 + k)/2 + jk/2$$
 V.

Power delivered by voltage source = S_v =

$$\frac{V_{src}}{\sqrt{2}} \frac{I_1^*}{\sqrt{2}} = \frac{1}{2} (5) (j3) = \frac{j15}{2} = j7.5 \text{ VA}$$

Power delivered by current source $S_l =$

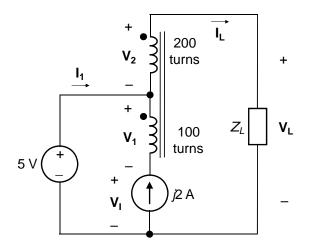
$$\frac{V_{I}}{\sqrt{2}} \frac{I_{SRC}^{*}}{\sqrt{2}} = \frac{1}{2} \left(\frac{15+k}{2} + \frac{jk}{2} \right) (-j2) = \frac{1}{2} \left(k - j(15+k) \right) = 0.5 - j8 \text{ VA}$$

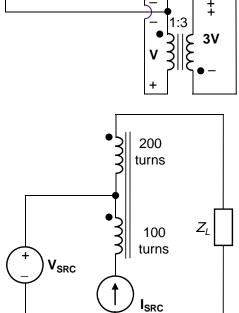
Total power delivered by sources =

$$\frac{1}{2}(j15+k-j15-jk)) = \frac{k}{2}(1-j)$$

As a check, $S_L =$

$$\frac{V_{Lm}}{\sqrt{2}}\frac{I_{Lm}^*}{\sqrt{2}} = \frac{1}{2}(-jk(1-j))(j) = \frac{k}{2}(1-j) \text{ VA}.$$





*j*5Ω |

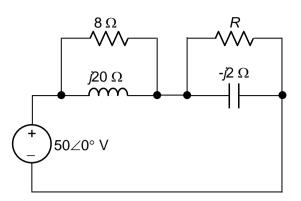
31

2**V**

2. In the circuit shown, the capacitance absorbs -200 VAR. Determine the average power dissipated in *R* if $R = 5 \Omega$.

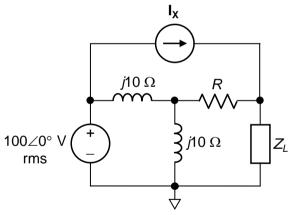
Solution: $Q = -BV_{rms}^2$, where V_{rms} is the rms voltage across *R* and *C*, and B = -1/X = 1/2 S. Substituting, $-200 = -\frac{1}{2}V_{rms}^2$, and $V_{rms} = 20$ V. It

follows that $P_R = \frac{V_{\rm rms}^2}{R} = \frac{400}{5} = 80$ W.



- 8. When a 9950 Ω resistance is connected in series with a D'Arsonval movement of unknown resistance and full-scale deflection current, a voltage of 1 V across the series combination gives a certain full-scale deflection. If an additional 10,000 Ω is connected in series with the combination, 2 V are required for full-scale deflection. Determine the resistance of the D'Arsonval movement.
- **Solution:** Let the resistance of the movement be R_m , its FSD current be I_{FSD} , and the FSD voltage with series resistance be V_{FSD} . Then $I_{FSD}(R + R_m) = V_{FSD}$, and $I_{FSD}(10,000 + R + R_m) = 2V_{FSD}$. It follows that $R + R_m = 10,000$, or $R_m = 10,000 R = 50 \Omega$.
- **11.** Determine Z_L for maximum average power delivered to it if $R = 5 \Omega$ and $I_x = k \angle -45^\circ$ where $k = \sqrt{2}$ A rms.

Solution: Z_{Th} is $(R + j5) \Omega$. Hence, Z_L for maximum power transfer is (R - j5) = (5 - j5) Ω .



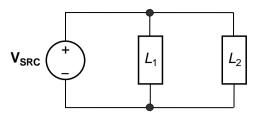
12. Determine the maximum average power delivered to Z_L in Problem 11, assuming that $R = 5 \Omega$ and I_X is as in Problem 11.

Solution: V_{Th} as seen by Z_L is determined from superposition as $\frac{j10}{j10+j10} \times 100 \angle 0^{\circ}$ +

 $(5 + j10 || j10)\mathbf{I}_{\mathbf{X}} = 50\angle 0^{\circ} + 5(1 + j)\mathbf{I}_{\mathbf{X}} = 50\angle 0^{\circ} + (5\sqrt{2}\angle 45^{\circ}) \times k\angle -45^{\circ} = 50 + 5\sqrt{2}k;$

$$P_{L\max} = \frac{\left(50 + 5\sqrt{2}k\right)^2}{4R_{Th}} = \frac{\left(50 + 5\sqrt{2}k\right)^2}{20} = \frac{\left(50 + 5\sqrt{2} \times \sqrt{2}\right)^2}{20} = \frac{\left(60\right)^2}{20} = 180 \text{ W}.$$

13. Load L_1 absorbs 15 kVA at 0.6 p.f. lagging, whereas Load L_2 absorbs 4.8 kW at 0.8 p.f. leading. If $V_{SRC} = 200 \angle 0^\circ$ V rms at f = 50 Hz, determine the capacitor that must be connected in parallel with L_1 and L_2 to have maximum magnitude of current through the source.



Solution: The reactive power absorbed by L_1 is 15×0.8 kVAR = 12 kVAR, whereas the reactive power absorbed by L_2 is $-\frac{4.8}{0.8} \times 0.6 = -3.6$ kVAR. For maximum magnitude of source current, the p.f. should be unity. The capacitor must add a reactive power of -(12 – 3.6) = -8400 VAR. hence, -8400 = $-\omega C \times |\mathbf{V}_{SRC}|^2$, or $C = \frac{8400}{100\pi |\mathbf{V}_{SRC}|^2} = \frac{84}{\pi |\mathbf{V}_{SRC}|^2} = \frac{84}{\pi |\mathbf{V}_{SRC}|^2}$

 $84/(\pi 2002) \equiv 0.67$ mF.

20. Determine the complex power delivered by
each source given that
$$V_{SRC} = 5\cos\omega t$$
, $I_{SRC} = -2\sin\omega t$,
and assuming $Z_L = k(1 - j)$ where $k = 1$.
Solution: The currents and voltages are as shown.
Equating mmfs: $100 \times 2 \angle 90^\circ + 200I_L = 0$, or $j2 = -2I_L$,
and $I_L = -j A$, $I_1 = I_L - j2 = -j3 A$.
 $V_L = Z_L I_L = -jk(1 - j) = -k(1 + j) \vee V_2 = V_L - 5 = -(k + (j - j)) = -(k + 5)/2 - jk/2 \vee V_1 = 5 - V_1 = (15 + k)/2 + jk/2 \vee V_2$.

Power delivered by voltage source = S_v =

$$\frac{V_{src}}{\sqrt{2}}\frac{I_1^*}{\sqrt{2}} = \frac{1}{2}(5)(j3) = \frac{j15}{2} = j7.5 \text{ VA}$$

Power delivered by current source S_{l} =

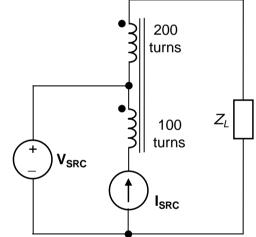
$$\frac{V_{I}}{\sqrt{2}} \frac{V_{SRC}}{\sqrt{2}} = \frac{1}{2} \left(\frac{15+k}{2} + \frac{jk}{2} \right) (-j2) = \frac{1}{2} \left(k - j(15+k) \right) = 0.5 - j8 \text{ VA}$$

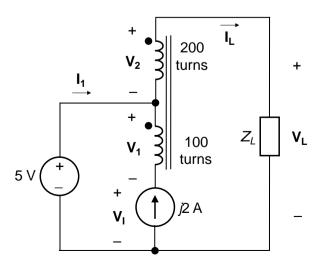
Total power delivered by sources =

$$\frac{1}{2}(j15+k-j15-jk)) = \frac{k}{2}(1-j)$$

As a check, $S_L =$

$$\frac{V_{Lm}}{\sqrt{2}}\frac{I_{Lm}^*}{\sqrt{2}} = \frac{1}{2}(-jk(1-j))(j) = \frac{k}{2}(1-j) \text{ VA}.$$

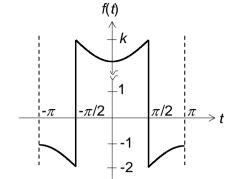




- **15.** The current waveform of the preceding problem is applied to a 2 Ω resistor in parallel with a very large capacitor. Determine the voltage across the parallel combination.
- **Solution:** The ac voltage will be negligibly small. The dc voltage is the dc component of current multiplied by *R*, or $V = 1 \times R = 2$ V.

16. The period of a periodic function f(t) is defined as:

 $f(t) = \cos(t + \pi) - 2, \qquad -\pi < t < -\pi/2$ $f(t) = -\cos(t) + k, \qquad -\pi/2 < t < +\pi/2$ $f(t) = \cos(t - \pi) - 2, \qquad \pi/2 < t < \pi$



Derive the trigonometric Fourier series expansion of f(t), assuming k = 3.

Solution:
$$a_{0} = \frac{1}{\pi} \left[\int_{0}^{\pi/2} (-\cos t + k) dt + \int_{\pi/2}^{\pi} (\cos(t - \pi) - 2) dt \right] =$$

 $\frac{1}{\pi} \left[k \int_{0}^{\pi/2} dt - 2 \int_{\pi/2}^{\pi} dt - \int_{0}^{\pi/2} \cos t dt - \int_{\pi/2}^{\pi} \cos t dt \right] = \frac{1}{\pi} \left[k \int_{0}^{\pi/2} dt - 2 \int_{\pi/2}^{\pi} dt \right] = \frac{1}{\pi} \left[\frac{k\pi}{2} - \pi \right] = \frac{k}{2} - 1.$
 $a_{n} = \frac{2}{\pi} \left[\int_{0}^{\pi/2} (-\cos t + k) \cos nt dt + \int_{\pi/2}^{\pi} (\cos(t - \pi) - 2) \cos nt dt \right] =$
 $\frac{2}{\pi} \left[-\int_{0}^{\pi} \cos t \cos nt dt + \int_{0}^{\pi/2} \cos nt dt - \int_{\pi/2}^{\pi} 2\cos nt dt \right] =$
 $\frac{2}{\pi} \left[-\frac{1}{2} \int_{0}^{\pi} \cos(n - 1) t dt - \frac{1}{2} \int_{0}^{\pi} \cos(n + 1) t dt + k \int_{0}^{\pi/2} \cos nt dt - 2 \int_{\pi/2}^{\pi} \cos nt dt \right] =$
 $-\frac{1}{\pi} \left[\frac{\sin(n - 1)t}{n - 1} + \frac{\sin(n + 1)t}{n + 1} \right]_{0}^{\pi} + \frac{2k}{n\pi} \left[\sin nt \right]_{0}^{\pi/2} - \frac{4}{n\pi} \left[\sin nt \right]_{\pi/2}^{\pi} =$
 $0 - 0 + \frac{2k}{n\pi} \sin \frac{n\pi}{2} - 0 - 0 + \frac{4}{n\pi} \sin \frac{n\pi}{2} = \frac{2(k + 2)}{n\pi} \sin \frac{n\pi}{2}.$ a_{n} is zero for even values, and the

odd harmonics alternate in sign. Thus,

$$f(t) = \left(\frac{k}{2} - 1\right)\frac{1}{2} + \frac{2(k+2)}{\pi}\left(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \frac{1}{7}\cos 7t + \dots\right)$$
$$= \frac{1}{2} + \frac{10}{\pi}\left(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \frac{1}{7}\cos 7t + \dots\right).$$

7. A D'Arsonval movement has a resistance of $R \Omega$ and a full-scale deflection of 100 μ A. Determine the shunt resistance that will result in a full-scale deflection of 150 μ A, assuming $R = 50 \Omega$. **Solution:** At full-scale deflection, the voltage drop across the movement and shunt is ($R = \Omega$)×(100 µA) = 100R µV. The shunt has to pass 50 µA, so its resistance is $R_{shunt} = 100R/50 = 2R = 100 \Omega$.

19. Determine X and R for maximum power transfer to R and *j*5 Ω 5Ω calculate this power. Assume $V_{SRC} = 4 \angle 0^{\circ} V$ 1:2 rms. V_{SRC} μ≥r 1:3 **Solution:** On open circuit, $V_{TH} = V = V_{SRC}$. On *j*5 Ω 5Ω short circuit, $\mathbf{V} = 0$ and $\mathbf{I}_{\mathbf{N}} = \mathbf{I} = \frac{\mathbf{V}_{\mathsf{SRC}}}{5(1+j)}$. $Y_N =$ V_{SRC} $\frac{1}{5(1+i)} = 0.1(1-i)$ S. For maximum power V_{Th} transfer, $G_L = 0.1$ S, or $R = 10 \Omega$, and $B_L = 0.1$ 3V S, or $X = -10 \Omega$. Under conditions of maximum power transfer, the current in R is $0.5|\mathbf{I}_{N}| = \frac{0.5 |\mathbf{V}_{SRC}|}{10}$ *j*5Ω | 5Ω and the power transferred is $\frac{0.25 |V_{sRC}|^2}{10} =$ 31 V_{SRC} $\frac{|\mathbf{V}_{\text{src}}|^2}{40} = \frac{16}{40} = 0.4 \text{ W}.$ 3V 14. A periodic current is shown, where over a А́ 6 period, $i = 6 + A \sin 2t$ $0 \le t \le \pi$ $i = -4 + A \sin 2(t - \pi)$ $\pi 0 \leq t \leq 2\pi$ 2π π $\rightarrow t$ Determine the rms value of *i* if A = 1 A. Α Solution: The waveform consists of three -4 components: i) a dc component of 1 A, ii) a

square wave of 5 V amplitude, and iii) a sinusoidal wave of amplitude A. It follows that the rms value is $I = \sqrt{1^2 + 5^2 + A^2/2} = \sqrt{26 + A^2/2} = \sqrt{26.5} = 5.15$ A.