2. Determine $R_{L}$ for maximum power transfer, and calculate the value of this power, assuming $V_{S}=12 \mathrm{~V}$ and $R=3 \Omega$.
$\rightarrow$ A.
$4 \Omega ; \quad 1 \mathrm{~W}$
B. $\quad 8 \Omega ; \quad 0.5 \mathrm{~W}$
C. $4 \Omega ; \quad 4 \mathrm{~W}$
D. $8 \Omega ; 2 \mathrm{~W}$

E. None of the above

12- Determine the maximum power transfer to the load RI, in the cottoning network. (The resistance are given in S)
a) 24 N
b) 36 W
c) 6 RL
(1) 36 RE

b) bone of the above

8. The source current in the circuit shown is $3 \cos (5000 t)$ A. What impedance should be connected across the terminals abb for maximum average power transfer.
a $10-20 j \Omega$
$\rightarrow$ b $\quad 20-10 j \Omega$
c. $10+20 j \Omega$
d. $10-20 j \Omega$

غ. None of the above.


## Problem 2

Find the value of the load impedance $\mathrm{Z}_{\mathrm{L}}$ such that maximum real power is delivered to it

$\rightarrow$ A) $3.0+\mathrm{j} 1.0 \Omega$
B) $1.75+\mathrm{j} 0.25 \Omega$
C) $1.5-\mathrm{j} 0.5 \Omega$
D) $3.0+\mathrm{j} 1.1 \Omega$
E) None of the above

## Problem 16



It is given that the load above has complex power $S==20+15 \mathrm{j} \mathrm{KVA}$. It is required to connect an element Z in parallel with the load so as to correct the power factor to unity (power factor= 1). The source voltage is $\mathrm{V}_{\mathrm{s}}=200 \angle 50 \mathrm{~V} \mathrm{rms}$ and the frequency is 60 Hz [that is $\left.\mathrm{Vs}=200 \cos \left(377 \mathrm{t}+50^{\circ}\right) \mathrm{V}(\mathrm{rms})\right]$. Determine the value and nature of this element Z .
$\rightarrow$ A) Capacitor with value $\mathrm{C}=994.7 \mu \mathrm{~F}$
B) Inductor with value $\mathrm{L}=7.07 \mathrm{mH}$
C) Capacitor with value $\mathrm{C}=663.13 \mu \mathrm{~F}$
D) Inductor with value $L=10.61 \mathrm{mH}$
E) None of the above.

## Problem 8 ( 14 pts)

Consider the circuit shown
a. Determine the value of the load $Z_{L}$ to maximize the average power absorbed by the load. (4 pts)

$$
\begin{aligned}
Z_{\text {src }} & =j 5.78+\frac{4(8-j 7)}{12-j 7} \\
& =3+j 5.2 \Omega ; \\
Z_{L m} & =3-j 5.2 \Omega .
\end{aligned}
$$


b. For the value obtained in (a), determine the average power developed by the voltage source and the average power absorbed by the load. (4 pts)

$$
\begin{aligned}
& \mathbf{V}_{\text {Th }}=12 \frac{8-j 7}{12-j 7}=9-j 1.74 \mathrm{~V} ; \\
& \mathbf{V}_{\text {Th }}=9.18 \mathrm{~V} ; \\
& \mathbf{I}_{\mathrm{L}}=\mathrm{V}_{\text {Th }} / 6=1.5-j 0.29 \mathrm{~A} ; \quad 12 \angle 0^{\circ} \mathrm{V} \\
& \left.\mathbf{V}_{1}=(3-j 0.58)\right)_{L}=4.68+j 0 \mathrm{~V} \\
& \mathbf{I}_{\text {SRC }}=\frac{12-4.68}{4}=1.83+j 0 \mathrm{~A} \\
& P_{\text {SRC }}=V_{1} I_{\text {SRC }}=12 \times 1.83 \cong 22 \mathrm{~W} ; P_{L}=\frac{(9.18)^{2}}{4 \times 3} \cong 7 \mathrm{~W} .
\end{aligned}
$$

c. For a purely resistive load, determine its value $R_{\max }$ for maximum power transfer and find the power absorbed by the load. (3 pts)

$$
\begin{aligned}
& R_{m}=\sqrt{(3)^{2}+(5.2)^{2}} \cong 6 \Omega ;\left|\|\left|=\left|V_{\mathrm{Th}}\right| / Z\right|=\frac{9.18}{\sqrt{(3+6)^{2}+(5.2)^{2}}}=0.883 \mathrm{~A}\right. \\
& P=\left|| |^{2} \times 6=4.68 \mathrm{~W} .\right.
\end{aligned}
$$

d. For a purely resistive load with $R_{L}=2 R_{\text {max }}\left(R_{\text {max }}\right.$ of part c), determine the power absorbed by the load. (3 pts)

$$
|I|=\left|V_{\mathrm{Th}}\right| /|Z|=\frac{9.18}{\sqrt{(3+12)^{2}+(5.2)^{2}}}=0.578 \mathrm{~A} ; P=|\||^{2} \times 12 \cong 4 \mathrm{~W}
$$

17. If a capacitor with impedance $Z_{2}$ is connected in parallel to a load $Z_{1}=300+j 450 \Omega$. What should be $\mathrm{Z}_{2}$ in ohms so that the equivalent load is purely resistive?
a) -928.6 j
b) -1112.5 j
$\longrightarrow$ c) -650 j
d) -750 j
e) None of the above
18. What is the power factor of the equivalent load of the previous question?
a) 0.8
b) 0.6
c) 0
$\rightarrow$ d) 1
e) None of the above
19. Find the maximum power dissipated in $R$ if: $I_{s}=2 m A, R_{1}=100 \mathrm{k} \Omega, V_{s}=50 \mathrm{~V}$.

a) $\mathrm{P}=12.5 \mathrm{~mW}$
b) $\mathrm{P}=1.25 \mathrm{~mW}$
c) $\mathrm{P}=50 \mathrm{~mW}$
$\rightarrow$ d) $\mathrm{P}=56.25 \mathrm{~mW}$
e) None of the above
-1- Two inductive loads of 0.88 KW and 1.32 KW at power factors of 0.8 and 0.6 respectively are connected in parallel across a $220-\mathrm{V}$ (rms), 50 Hz supply. Calculate the total current taken by this combination.
a. 1 A
b. 14.86 A
c. 10.86 A
d. 15.45 A
e. None of the above
-2- For the previous problem, find the value of capacitance in microfarads, to be connected in parallel with the loads to bring the combined power factor to 0.9 lagging.
$\longrightarrow$ a. 89
b. 35.8
c. 25.6
d. 44.5
e. None of the above
-6- Two impedances $Z=(2+\mathrm{j} 4) \Omega$ and $Z^{\prime}=\mathrm{R} \Omega$ are connected in parallel. Find R so that the power factor of the circuit is 0.9 lag .
a. $1.3 \Omega$
万. $3.2 \Omega$
c. $2.4 \Omega$
d. $3 \Omega$
e. None of the above
20. Find the maximum average power given that $R_{L}$ is adjusted for maximum power transfer.
A. 50 W
B. 10 W
C. 100 W
D. 25 W
E. None of the above


## 8\%

4. $R_{L}$ and the turns ratio of the ideal transformer can be varied over an arbitrary range. Determine $R_{L}$ for maximum power transfer to it.

A. $10 \Omega$
B. $20 \Omega$
C. $30 \Omega$
D. $40 \Omega$
E. None of the above

Solution: To have the reactances add to zero, the transformer turns ratio must be 2, primary-to-secondary. Hence $R_{L}=4 \times 10=40 \Omega$.

-3- The load impedance $Z_{L}$ for the circuit shown is adjusted until maximum average power is delivered to the load. Find this maximum power.

a. 5 W
b. 25.34 W
c. 296.8 W
d. 7.81 W
e. None of the above
11. Determine the frequency at which maximum power is dissipated in the $10 \Omega$ resistor, assuming $L=1 \mathrm{H}$.
Solution: $\frac{1}{\omega C}=\frac{1}{\omega} \Omega$. Maximum power is
dissipated in the $10 \Omega$ resistor when $X_{L}=-X_{C}$,

which gives $\omega L=\frac{1}{\omega}$, or $\omega=\frac{1}{\sqrt{L}} \mathrm{rad} / \mathrm{s}$.

## Eng. $\alpha$ Arch. LIDray

4. Consider the following circuit where V2 is a current dependent source of voltage. $R_{L}$ is a variable pure resistive load. Calculate the value of $R_{L}$ that dissipates the maximum average power. Given $X=1 \Omega, R=2 \Omega, k=1$
A. $\sqrt{ } 5 \Omega$
B. $1 / \sqrt{ } 5 \Omega$
C. $2 / \sqrt{ } 5 \Omega$

D. $2 \Omega$
E. None of the above

> arge u fuli. Lividy
7. Given that the complex power absorbed by the inductive branch is $12+j 16 \mathrm{VA}$, find the smallest $C$ that gives unity power factor at terminals $a b$, assuming $\omega=1 \mathrm{rad} / \mathrm{s}$.
A. 0.2 F
B. 0.1 F
C. 0.05 F
D. 0.15 F

E. None of the above
18. Find the wattmeter reading of the circuit below. Where $\mathrm{C}=1 / 2 \mathrm{~F}$

$$
V_{m s}=150<0^{\circ}, v
$$


(a) $\mathrm{P}=387.93 \mathrm{~W}$
(b) $\mathrm{P}=412.84 \mathrm{~W}$
(c) $\quad \mathrm{P}=445.54 \mathrm{~W}$
(d) $\mathrm{P}=279.4 \mathrm{~W}$
(e) None of the above
9. What should be the value of resistance $\mathrm{R}_{\mathrm{L}}$ for maximum power to be transferred to it?

a. $R_{L}=5 K \Omega$
b. $R_{\mathrm{L}}=10 \mathrm{~K} \Omega$
c. $\mathrm{R}_{\mathrm{L}}=7.62 \mathrm{~K} \Omega$
d. $\mathrm{R}_{\mathrm{L}}=7.07 \mathrm{~K} \Omega$
e. None of the above
5. An electric motor draws an active power of 100 kW at 0.8 p.f lagging from a $240 \mathrm{~V}, 60 \mathrm{~Hz}$ source. This motor is connected in parallel to another load of $0.1+j 0.4 \Omega$. What is the size of the parallel connected capacitor needed to raise the total power factor to 0.95 lagging.
a. 6.27 mF
b. 4.25 mF
c. 7.68 mF
d. 2.88 mF
e. None of the above

2. A $1 \Omega$ resistor is connected in parallel with a d'Arsonval movement having a full scale deflection of 1 mA . If a 40 mA current produces a deflection that is $80 \%$ of full scale, determine the resistance of the d'Arsonval movement.
a) $58 \Omega$
$\longrightarrow$ b) $49 \Omega$
c) $37 \Omega$
d) $76 \Omega$
e) None of the above
8. A 300-v voltmeter that draws 2 mA current for full-scale reading is used to measure the voltage across the $50-\mathrm{KQ}$ resistor of Figure 6 . The voltmeter reading is:
a. 60 V
b. 120 V
c. 40 V
(i.) 90 V

None of the above
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Fig. 6
15. A $500,0.1 \mathrm{~A}$ d'Arsonval meter movement is used in a voltmeter circuit (Figure 13). Determine the voltmeter reading across the terminals $x-y$ on a iull-scale of 30 V .
8. $\quad 26.49 \mathrm{~V}$
b. 32 V
31.79 V

36 V
(E) None of the above

10. A load resistance in the range of $[1,5] \Omega$ is to be connected across the terminals $a, b$ in such a way that maximum power is delivered to it. Determine the power dissipated by $R$.

a) 10.58 W
b) 4.13 W
c) 5.95 W
d) 8.26 W
e) none of the above
13. Find the maximum power transfer in the circuit below, where $\mathrm{R}=2 \Omega$

(a) 13.44 W
(b) 10.08 W
(c) 12.1 W
(d) 8.34 W
(e) None of the above

7\%
4. For what value of $R$ is maximum power transferred to $R_{L}$ ?
A. $10 \Omega$
B. $15 \Omega$
C. $20 \Omega$
D. Infinite resistance
E. None of the above


7\%
5. A circuit N has an open-circuit voltage of 15 V between terminals ab, and an unknown source resistance $R_{\mathrm{S}}$. A voltmeter across ab reads 12 V when $R_{L}=10 \mathrm{k} \Omega$ and 10 V when $R_{L}=40 / 9 \mathrm{k} \Omega$. Determine $R_{S}$ and $R_{V}$, the resistance of the voltmeter.
A. $R_{S}=2 \mathrm{k} \Omega, R_{V}=40 \mathrm{k} \Omega$

B. $R_{S}=2 \mathrm{k} \Omega, R_{V}=80 \mathrm{k} \Omega$
C. $R_{S}=4 \mathrm{k} \Omega, R_{V}=40 \mathrm{k} \Omega$
D. $R_{S}=4 \mathrm{k} \Omega, R_{V}=80 \mathrm{k} \Omega$
E. None of the above

17\%
7. Determine $R$ for maximum power transfer to it and the value of this power.


Solution: When $R$ is replaced by an open circuit, the current source is set to zero. The circuit becomes as shown. $V_{T h}=V_{a b}=$ $15 \times \frac{20}{30}=10 \mathrm{~V}$.

When a source $V_{T}$ is applied, with the 15 V short circuited, and $I_{T}$ as shown, the polarity of the current source is reversed. From KVL: $V_{T}=5 I_{T}$ $(1+2 \| 1)$. This gives $\frac{V_{T}}{I_{T}}=R=R_{T h}=\frac{25}{3} \Omega$. Max power transferred is $\frac{100}{4 \times 25 / 3}=3 \mathrm{~W}$.


## Problem 3

Consider the periodic signal $\mathrm{V}_{\mathrm{s}}(\mathrm{t})$ shown below. Assume this signal is applied to the circuit in the figure below, find an expression for the voltage $V_{o}(t)$ across the terminals $a, b$ as shown.

$\rightarrow$ A) $\quad V_{o}(t)=\frac{V_{m}}{8}+\frac{V_{m}}{2 \pi} \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{2}}{n} \cos n \omega_{o} t$
B) $V_{0}(t)=\frac{V_{m}}{8}+\frac{V_{m}}{4 \pi} \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{4}}{n} \cos n \omega_{o} t$
C) $V_{4}(t)=\frac{V_{m}}{4}+\frac{V_{m}}{2 \pi} \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{2}}{n} \cos n \omega_{o} t$
D) $V_{4}(t)=\frac{V_{m}}{4}+\frac{V_{m}}{2 \pi} \sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{2}}{n} \cos n \omega_{o} t$
E) None of the above
-1- Consider the two signals :

$$
\begin{aligned}
& \mathrm{F}(\mathrm{t})=2+3 \operatorname{Cos}(100 \pi \mathrm{t})+4 \operatorname{Cos}(200 \pi \mathrm{t})+6 \operatorname{Cos}(400 \pi \mathrm{t}) \\
& \mathrm{G}(\mathrm{t})=\frac{\operatorname{Cos}(100 \pi \mathrm{t}) \cdot \operatorname{Sin}(300 \pi \mathrm{t})-\operatorname{Sin}(100 \pi \mathrm{t}) \cdot \operatorname{Cos}(300 \pi \mathrm{t})}{\operatorname{Sin}(200 \pi \mathrm{t})}
\end{aligned}
$$

Find the period of each of the signals, TF and TG. (p25)
a) $\mathrm{TF}=1 / 100 \quad \mathrm{TG}=1 / 100$
b) $\mathrm{TF}=1 / 200 \quad \mathrm{TG}=1 / 300$
c) $\mathrm{TF}=1 / 50 \quad \mathrm{TG}=1 / 50$
d) $T F=7 / 200 \quad \mathrm{TG}=1 / 200$
e) None of the above $T F=1 / 50, G(t)=1$
-2- The signal $F(t)$ given in (1) is an approximation of the real signal $f(t)$ with average power equal to 50Watt.
What is the \%average power error in the approximation?(take $\mathrm{R}=1 \Omega)(225)$
(a) $31 \%$
b) $35 \%$
c) $65 \%$
d) $39 \%$
e) None of the above
-3- Consider the trigonometric Fourier series representation of $f(t)$ as given over the interval $(-2,2): f(t)=t+1-1 \leq t \leq 0$

$$
\begin{array}{cl}
-t+1 & 0 \leq t \leq 1 \\
0 & \text { elsewhere }
\end{array}
$$

The Fourier series is also a representation of the periodic signal $F(t)$ obtained by repeating $f(t)$ periodically. The period of $F(t)$ is:
a) 3
b) 2
c) 1
d) 4
e) None of the above
-4- Two periodic functions of period 6 seconds each are given by:

$$
\begin{array}{rlrl}
f(t)=-t & -3<t \leq 0 & g(t)=1 & 0<t<3 \\
t & 0 \leq t<3 & -1 & 3<t<6
\end{array}
$$

Find the ratio of the amplitude of the $3^{\text {rd }}$ harmonic present in $f(t)$ to that present in $g(t)$. $p 25$ )
(a) $1 / \pi$
b) $3 / \pi$
c) $\pi / 3 \pi$
d) $\pi$ e) None of the above
-5- Calculate the rms value of the current flowing in the circuit shown. (p26)
a) 8.5 A
b) 54.85 A
(c) 42.58 A
d) 25 A
e) None of the above
-6- For the wave shown, calculate $A_{0}$, $A_{5}$ and $B_{5}$ ( ${ }^{26}$ 26)
a) $\mathrm{A}_{0}=1.5, \mathrm{~A}_{5}=1.5$
b) $A_{5}=1.15$
c) $\mathrm{B}_{5}=1.5$
d) $\mathrm{A}_{5}=1.5, \mathrm{~B}_{5}=1.15$
e) None of the above $A_{0}=1.5, A_{5}=1.15, B_{5}=0$

-7- Find the average power in a resistance $R=2 \Omega$, if the voltage is $v=32 \sin (3 t) \operatorname{Cos}^{2}(t / 2)(p 26)$
a) 256 W
b) 512 W
c) 192 W d) 96 W
e) None of the above
-8- In a circuit : $v(t)=5 \operatorname{Sin}(t)+10 \operatorname{Sin}(3 t)$,

$$
I(t)=7 \sin (2 t)+50 \sin (8 t)
$$

What is the average power ? (p26)
a) 267.5 W
(b) 0 W
c) 250 W
d) 35 W
e) None of the above
-9- Two periodic voltages $\mathrm{V} 1(\mathrm{t})$ and $\mathrm{V} 2(\mathrm{t})$ of the same period ( $\mathrm{T}=2 \mathrm{~s}$ ) are applied to the circuit. Find the current in this circuit due to the first harmonic. (p26)

a) $12 \operatorname{Cos}(\pi t)$
b) $12 \operatorname{Sin}(\pi t)$
c) $12 \operatorname{Cos}(\pi t)+12 \operatorname{Sin}(\pi t)$
d) 0
e) None of the above
-10- The function $e^{x}(0<x<1)$ is to be represented by a cosine series, find $a_{2}$. (p2b)
a) -0.684
b) -0.0827
c) 0.0848 d) 0.0424
e) None of the above
-11-Consider the wave form $f(t)$. Find the amplitude and phase of the $3^{\text {rd }}$ harmonic components of this waveform. (p27)

a) $C_{3}=5.1 \quad \theta_{3}= \pm 90^{\circ}$
b) $\mathrm{C}_{3}=2.57 \quad \theta_{3}= \pm 90^{\circ}$
c) $C_{3}=4.8 \quad \theta_{3}= \pm 180^{\circ}$
d) $C_{3}=5.1 \quad \theta_{3}= \pm 180^{\circ}$
(e) None of the above
-12- A rectifier system with input $f(t)$ and output $g(t)$ is described by: $g(t)=|f(t)|$. For an input of $f(t)=(\pi / 4) \operatorname{Sin}\left(\omega_{0} t\right)$, the coefficient of the exponential Fourier series of $g(t)$ with $n$ even is: (p27)
(a) $1 / 2\left(1-n^{2}\right)$
b) $1 /\left(1-n^{2}\right)$
c) $2 /\left(1-n^{2}\right)$
d) $1 /\left(\pi-\pi n^{2}\right)$
e) None of the above
-13- Let the signal $f(t)$ be a signal defined between $-\pi$ and $\pi$. $F(t)$ is zero outside the following exponential Fourier series: (p27)

$$
\sum_{n=-\infty}^{\infty} C_{n} \cdot e^{\left(\frac{j n}{2}\right)}
$$

The series represents the periodic extension of $f(t)$ with period $T$. Find $T$.
a) $8 \pi$
b) $2 \pi$
c) $6 \pi$
(d) $4 \pi$
e) None of the above
-14- Consider the following signal :
$F(t)=2 \operatorname{Cos}(100 \pi t)+3 \operatorname{Cos}(300 \pi t)+6 \operatorname{Cos}(500 \pi t)+9 \operatorname{Sin}(300 \pi t)$
Find the coefficients of the exponential Fourier series of $F(t) .(p 27)$
a) $\mathrm{C}_{1}=\mathrm{C}_{-1}=1 ; \mathrm{C}_{3}=1.5-4.5 \mathrm{j} \mathrm{C}_{-3}=1.5+4.5 \mathrm{j} ; \mathrm{C}_{5}=\mathrm{C}_{-5}=3$
b) $\mathrm{C}_{1}=\mathrm{C}_{-1}=-1 ; \mathrm{C}_{3}=4.5-1.5 \mathrm{j} \quad \mathrm{C}_{-3}=4.5+1.5 \mathrm{j} ; \mathrm{C}_{5}=\mathrm{C}_{-5}=-3$
c) $\mathrm{C}_{1}=\mathrm{C}_{-1}=2 ; \mathrm{C}_{3}=2.5-4.5 \mathrm{j} \quad \mathrm{C}_{-3}=2.5+4.5 \mathrm{j} ; \mathrm{C}_{5}=\mathrm{C}_{-5}=3$;
d) $\mathrm{C}_{1}=\mathrm{C}_{-1}=2 ; \mathrm{C}_{3}=2.5-4.5 \mathrm{j} \quad \mathrm{C}_{-3}=2.5+4.5 \mathrm{j} ; \mathrm{C}_{5}=\mathrm{C}_{-5}=3$;
e) None of the above

1. Find the mm value of $v(t)$ in Fig. 1 over the time interval $(0,5) .(p>0)$
A. $\mathrm{v}_{\mathrm{ms}}=0.54$
B. $\mathrm{v}_{\mathrm{rms}}=0.98$

- C. $\mathrm{v}_{\mathrm{rms}}=0.86$
D. $\mathrm{v}_{\mathrm{rms}}=0.68$
E. None of the above


Figure 1.
10. Find the expression for the Fourier coefficients
$\mathrm{C}_{\mathrm{n}}$ for the periodic function shown in Fig. 9. (p72).
A. $1 / 2 \pi \mathrm{n}, \mathrm{n} \neq 0 ; 1 / 2, \mathrm{n}=0$
B. $\mathrm{j} / \pi \mathrm{n}, \mathrm{n} \neq 0 ; \quad 0, \mathrm{n}=0$
$\rightarrow$ C. $\mathrm{j} / 2 \pi \mathrm{n}, \mathrm{n} \neq 0 ; \quad \mathrm{j} / 2, \mathrm{n}=0$
D. $j / 3 \pi n, n \neq 0 ; \quad 1, n=0$
E. None of the above


Figure 9.
11. A periodic function is represented by:

$$
v(t)=\sum_{n=-\infty}^{+\infty} V_{n} e^{j 200 \pi n t}
$$

Fig. 10 shows the plot of the magnitude of the coefficients Vn. Find the average and the fundamental frequency of $v(t) .(p 72)$
A. $2 ; 1 \mathrm{~Hz}$
B. $5 ; 10 \mathrm{~Hz}$
C. $0 ; 10 \mathrm{~Hz}$
$\rightarrow$ D. $5 ; 100 \mathrm{~Hz}$
E. None of the above


Figure 10.
14. Calculate the power dissipated in the resistor in Fig. 12 if $\mathrm{v}_{1}(\mathrm{t})=10$ cost and $\mathrm{v}_{2}(\mathrm{t})=10 \cos 3 \mathrm{t} .(p>3)$

4. A series RL circuit in which $\mathrm{R}=5 \Omega$ and $\mathrm{L}=20 \mathrm{mH}$ has an applied voltage $\mathrm{v}=100+$ $50 \sin \omega t+25 \sin 3 \omega t \mathrm{~V}$, with $\omega=500 \mathrm{rads} / \mathrm{s}$. Find the instantaneous current. ( $p / 0 G$ )
a. $i=20+4.47 \sin (\omega t+63.4)+0.822 \sin (3 \omega t+80.54), A$
b. $i=20+4.47 \sin (\omega t-63.4)+0.822 \sin (3 \omega t-80.54)$, A
c. $i=8.96+4.47 \sin (\omega t-63.4)+0.822 \sin (3 \omega t-80.54), A$
d. $i=\sin (\omega t-63.4)+0.822 \sin (3 \omega t-80.54)$, $A$
e. None of the above
5. Determine the power dissipated in the resistor of problem 4. $p(106)$
a. $\sim 50.1 \mathrm{~W}$
b. $\sim 51.79 \mathrm{~W}$
c. $\sim 2000 \mathrm{~W}$
d. $\sim 2053 \mathrm{~W}$
e. None of the above
11. The figure below shows the triangular waveform of a voltage source operating at frequency $f=1 \mathrm{kHz}$. Find the amplitudes of the fundamental $\left(l_{1}\right)$ and the second order harmonic $\left(I_{2}\right)$ current that flows through an inductor of value $\mathrm{L}=1 \mathrm{mH}$ when it is supplied by this source.. (Answers are rounded to 2 digits after the decimal point) (p106)
a) $\mathrm{I}_{1}=2.31 \mathrm{~A}, \mathrm{I}_{2}=1.10 \mathrm{~A}$
b) $\mathrm{I}_{1}=1.34 \mathrm{~A}, \mathrm{I}_{2}=0 \mathrm{~A}$
c) $l_{1}=0 \mathrm{~A}, \mathrm{l}_{2}=1.10 \mathrm{~A}$
d) $l_{1}=1.29 \mathrm{~A}, 1_{2}=0 \mathrm{~A}$
e) None of the above

8. The following spectrum is the frequency representation of which Fourier function:

a. $f(t)=\frac{4 V}{\pi} \sin \omega t+\frac{4 V}{3 \pi} \sin 3 \omega t+\frac{4 V}{5 \pi} \sin 5 \omega t+\ldots$
b. $f(t)=\frac{V}{2}+\frac{4 V}{\pi} \sin \omega t+\frac{4 V}{3 \pi} \sin 3 \omega t+\frac{4 V}{5 \pi} \sin 5 \omega t+\ldots$
c. $f(t)=\frac{V}{2}+\frac{4 V}{\pi^{2}} \cos \omega t+\frac{4 V}{(3 \pi)^{2}} \cos 3 \omega t+\frac{4 V}{(5 \pi)^{2}} \cos 5 \omega t+\ldots$
d. $f(t)=\frac{V}{8}+\frac{4 V}{\pi^{2}} \sin \omega t+\frac{4 V}{(3 \pi)^{2}} \sin 3 \omega t+\frac{4 V}{(5 \pi)^{2}} \sin 5 \omega t+\ldots$
$\rightarrow$ e. None of the above
4. A complex waveform of RMS value of 240 V has $20 \% 3$-rd harmonic content, $5 \% 5$-th harmonic content and $2 \% 7$-th harmonic content. Find the RMS value of the 3 -rd and 7 -th harmonics respectively. ( $p 139$ )
A. $11.5 \mathrm{~V}, 4.6 \mathrm{~V}$
B. $7.6 \mathrm{~V}, 1.3 \mathrm{~V}$
C. $47 \mathrm{~V}, 4.7 \mathrm{~V}$
D. $30 \mathrm{~V}, 3.2 \mathrm{~V}$
E. None of the above
11. A voltage $v(t)$ is applied to a $5 \Omega$ resistor. $v(t)$ can be written as:

$$
v(t)=1-\Sigma_{n=1}^{\infty}\left(1 / n^{2}\right) \cos (500 n t)
$$

Estimate the Power dissipated in the resistor using the first four non-zero terms of $\mathrm{v}(\mathrm{t})$. (p141)
a. 2.36 W
b. 0.31 W
c. 1.25 W
d. 0.95 W
e. None of the above
18. The periodic voltage $v_{1}$ is applied to the circuit shown, the reactance of $C$ at the frequency of the fundamental being much smaller than $10 \Omega$. Determine the power dissipated in the circuit. ( $p 144$ )
A. 13.33 W
B. 8.33 W
C. 6.67 W

E. None of the above
20. The current through a $1 \mu \mathrm{~F}$ capacitor is $2 \cos ^{2} 100 \pi t \mathrm{~mA}$, where $t$ is in s . Determine the period of the voltage across the capacitor.
A. 25 ms
B. 50 ms
C. 100 ms
D. 200 ms
E. None of the above
4. The periodic voltage waveform shown below is applied across a $10 \Omega$ resistor. Determine the average power dissipated in the resistor. (p/5>)

$$
\hat{V g}(t), V
$$

a. 8.8 W
b. 88 W
c. 77.4 W
d. 4.4 W
e. None of the above

9. A half-wave rectified waveform $f_{l w}(t)$ of frequency 50 Hz and having $A=10 \mathrm{~V}$ is applied to the circuit shown, where the reactance of $C$ is negligible at 50 Hz .

Determine the total power dissipated in the circuit. ( $p / 58$ )
A. 0.83 W
B. 1.49 W
$\rightarrow$ C. 1.82 W
D. 2.5 W
E. None of the above


4. A voltage having the waveform of the figure of Problem 9 below, with $A=8 \mathrm{~V}$ and $T$ $=1 \mathrm{~s}$ is applied to a coil having a resistance of $4 \Omega$ and an extremely large inductance. Determine the average power dissipated in the coil. ( $p$ /70)
A. 1.56 W
B. 1 W
C. 3.28 W
D. 4 W
E. None of the above

## Problem 9

Derive the trigonometric form of the FSE of the waveform
shown
Solution: The function is even, and $a_{0}=C_{0}=$

$$
\frac{1}{T} \times A \times \frac{T}{4}=\frac{A}{4} ; a_{n}=\frac{4 A}{T} \int_{0}^{T / 4}\left(-\frac{4}{T} t+1\right) \cos n \omega_{0} t d t
$$

$$
=\frac{4 A}{T}\left[-\frac{4}{T} \frac{1}{n^{2} \omega_{0}^{2}} \cos n \omega_{0} t-\frac{4}{T} \frac{t}{n \omega_{0}} \sin n \omega_{0} t-\frac{1}{n \omega_{0}} \sin n \omega_{0} t\right]_{0}^{T / 4}
$$

$$
=\frac{16 A}{T^{2} n^{2} \omega_{0}^{2}}\left[1-\cos \frac{n \pi}{2}\right]=\frac{4 A}{\pi^{2} n^{2}}\left(1-\cos \frac{n \pi}{2}\right) . \text { Hence, }
$$

$$
f(t)=\frac{A}{4}+\frac{4 A}{\pi^{2}}\left(\cos \omega_{0} t+\frac{1}{2} \cos 2 \omega_{0} t+\frac{1}{9} \cos 3 \omega_{0} t+\frac{1}{25} \cos 5 \omega_{0} t+\ldots\right)
$$

6. Consider a periodic function $f(t)$, described by the following sequence during one period of time:

$$
\begin{array}{lll}
\mathrm{f}(\mathrm{t})=0 & \text { for } & -4 \leq t<-3 \\
\mathrm{f}(\mathrm{t})=-\mathrm{Vm} & \text { for } & -3 \leq t<-1 \\
\mathrm{f}(\mathrm{t})=0 & \text { for } & -1 \leq t<+1 \\
\mathrm{f}(\mathrm{t})=+\mathrm{Vm} & \text { for } & +1 \leq t<+3 \\
\mathrm{f}(\mathrm{t})=0 & \text { for } & +3 \leq t \leq+4
\end{array}
$$

where $\mathrm{Vm}=20$. Find the amplitude " $A$ " of the third order harmonic in the Fourrier series, expressed by $A \cos (3 \omega t-\Theta)$. (p/7b)
A. $A=3$
B. $A=4$
C. $A=5$
D. $A=6$
E. None of the above

Solution: The function is as shown. It is odd and quarterwave symmetric. Hence, $a_{0}=0$ and $b_{3}=\frac{8}{T} \int^{2} V_{m} \sin 3 \omega_{0} t d t$, where $T=8$ and $\omega_{0}=\frac{2 \pi}{8}=$

$\frac{\pi}{4}$. Hence, $b_{3}=\frac{8 V_{m}}{3 \omega_{0} T}\left[-\cos 3 \omega_{0} t\right]_{1}^{2}=\frac{160}{6 \pi}\left[-\cos \frac{3 \pi}{4}+\cos \frac{\pi}{2}\right]=\frac{80}{3 \pi \sqrt{2}}=6.0$
11. A voltage $5 \sin \omega_{0} t \mathrm{~V}$ applied to a given resistor dissipates 5 W . What is the power dissipated by a voltage $5\left|\sin \omega_{0} t\right| \mathrm{V}$ applied to the same resistor? (p17)
A. 5 W
B. $5 \sqrt{2} \mathrm{~W}$
C. $5 / \sqrt{2} \mathrm{~W}$
D. 10 W
E. None on the above

Solution: The two waveforms have the same rms value and would therefore dissipate the same power in a given resistor.
D. None of the above

Solution: Mean square is $\frac{64 \times 2+100 \times 3+64 \times 3+100 \times 2}{10}=82$.

$$
P=\frac{82}{10}=8.2 \mathrm{~W}
$$



1. in the circuit shown, each source is $15 \cos 10 t \mathrm{~V}$. The power dissipated in $R$ is 50 W . If the frequency of one of the sources is doubled, the power dissipated in $R$ is:
A. 100 W
B. 50 W
C. 25 W
D. 12.5 W

E. None of the above.

Solution: The current due to each source is $\frac{1}{2}\left(\frac{15}{1.5}\right) \cos 10 t=5 \cos 10 t A$. The power is $\left(\frac{5}{\sqrt{2}}\right)^{2}=12.5 \mathrm{~W}$. the power dissipated due to both sources is 25 W .

2. $f_{2}(t)$ is the function $f_{1}(t)$ lowered by 1 unit, as shown. If $F_{1 \mathrm{rms}}$ and $F_{2 \mathrm{rms}}$ are the rms values of


Solution: The AC components of $f_{1}(t)$ and $f_{2}(t)$ are the same. The DC component of $f_{1}(t)$ is larger than that of $f_{2}(t)$. Hence, $F_{1 \mathrm{rms}}>F_{2 \mathrm{rms}}$.
5. The Fourier coefficients $a_{k}$ and $b_{k}$ for the periodic function shown are:
A. $a_{k}=0$ for all $k ; b_{k}=0$ for $k$ odd and is non-zero for $k$ even
B. $b_{k}=0$ for all $k ; a_{k}=0$ for $k$ even and is non-zero for $k$ odd
C. $b_{k}=0$ for all $k ; a_{k}=0$ for $k=0, a_{k}=0$ for $k$ odd
 and is non-zero for $k$ even
D. $a_{k}=0$ for all $k ; b_{k}=0$ for $k$ even and is non-zero for $k$ odd
E. None of the above.

Solution: The function is odd, half-wave symmetric. Its average is zero; it contains no cosine terms, only odd sine terms.
11. The current through an inductor of 1 H is given by the periodic triangular wave. The amplitude of the fundamental component of the voltage across the inductor is:
A. $4 I_{p}$
B. $8 \rho_{p}$
C. $16 I_{p}$

D. $32 I_{p}$
E. None of the above.

Solution: $v=L \frac{d i}{d t}=1 \times \frac{2 I_{p}}{1 / 4 \pi}=8 \pi t_{p}$, which is the amplitude of the square waveform representing $v$. the amplitude of the fundamental is $\frac{4 \times 8 \pi 1_{p}=321_{p}}{\pi}$.
15. The voltage and current at the terminals of a circuit are:

$$
\begin{aligned}
& v=15+400 \cos 500 t+100 \sin 1500 t \mathrm{~V} \\
& i=2+5 \sin \left(500 t+60^{\circ}\right)+3 \cos \left(1500 t-15^{\circ}\right) \mathrm{A}
\end{aligned}
$$


$3 \%$ a) Calculate the average power delivered to the circuit.

$$
\begin{aligned}
P & =V_{d c} I_{d c}+\sum_{n=1}^{3} \frac{V_{m} I_{m}}{2} \cos \left(\theta_{v n}-\theta_{i n}\right)=15 \times 2+\frac{1}{2} \times 400 \times 5 \cos \left(30^{\circ}\right)+\frac{1}{2} \times 100 \times 3 \times \cos \left(-75^{\circ}\right) \\
& =934.85 \mathrm{~W}
\end{aligned}
$$

3\%
b) Calculate the rms value of $v$.
$V_{\text {rms }}=\sqrt{(15)^{2}+\frac{(400)^{2}}{2}+\frac{(100)^{2}}{2}=291.93} \mathrm{~V}$
c) Calculate the rms value of $i$.
$I_{\text {rms }}=\sqrt{(2)^{2}+\frac{(5)^{2}}{2}+\frac{(3)^{2}}{2}=4.58} \mathrm{~A}$
12. For $n=1,2,3, \ldots$, the function shown has:
A. $a_{n}$ and $b_{n}$ nonzero for all $n$
B. $a_{n}$ and $b_{n}$ are zero for even $n$
C. $a_{n}$ and $b_{n}$ are zero for odd $n$
D. $a_{n}=0$ for all $n$
E. $b_{n}=0$ for all $n$

Solution: When the dc value is removed, the ac
 component has half-wave symmetry but is neither even nor odd. Hence, $a_{n}$ and $b_{n}$ are zero for even $n$.
13. Determine the total power dissipated if $v_{l}$ is a full-wave rectified waveform given by: $v_{l}=6|\sin (500 t)| V$.
Solution: $\frac{1}{\omega C}=\frac{1}{500 \times 100}=2 \times 10^{-5} \ll 5$ ohms; $V_{d c}=\frac{2 V_{m}}{\pi}$; $P_{d c}=\frac{4 V_{m}^{2}}{10 \pi^{2}}=0.04053 V_{m}^{2} ;$

$\frac{V_{m}^{2}}{2}=\frac{4 V_{m}^{2}}{\pi^{2}}+V_{a c}^{2} ; V_{a c}^{2}=V_{m}^{2}\left(\frac{1}{2}-\frac{4}{\pi^{2}}\right)=0.09472 V_{m}^{2} ; P_{a c}=\frac{0.09472 V_{m}^{2}}{5} 0.01894 V_{m}^{2} ;$
$P=0.05947 V_{m}^{2}$.
14. A period of a periodic function $f(t)$ is given by: $K(4+$ $2 \sin t), 0<t<2 \pi$. Determine the rms value of $f(t)$, if $K=$ 0.5 .

Solution: The square of $f(t)$ is $K^{2}\left(16+16 \sin t+4 \sin ^{2} t\right)=$
$K^{2}(16+2+16 \sin t-2 \cos 2 t)$. The area under the square is
 $\int_{0}^{2 \pi} K^{2}(16+2+16 \sin t-2 \cos 2 t) d t=36 \pi K^{2}$; the mean square is $\frac{36 \pi K^{2}}{2 \pi}=18 K^{2}$ and the rms value is $3 \sqrt{2} K$.
16. Derive the trigonometric Fourier expansion of the given periodic function $f(t)$.


Solution: Since $f(t)$ is odd, $a_{0}=0=a_{n} ; T=2, \omega_{0}=2 \pi / T=\pi, f(t)=t+1$;
$b_{n}=\frac{4}{T} \int_{0}^{T / 2} f(t) \sin \left(n \omega_{0} t\right) d t=2 \operatorname{lm}\left[\int_{0}^{T / 2} f(t) e^{j n \omega_{0} t} d t\right]=2 \operatorname{lm}\left[\frac{t e^{j n \omega_{0} t}}{j n \omega_{0}}-\frac{e^{j n \omega_{0} t}}{\left(j n \omega_{0}\right)^{2}}+\frac{e^{j n \omega_{0} t}}{j n \omega_{0}}\right]_{0}^{1}=$
$2 \operatorname{lm}\left[\frac{e^{j n \pi}}{j n \pi}+\frac{e^{j n \pi}}{(n \pi)^{2}}+\frac{e^{j n \pi}}{j n \pi}-0+\frac{1}{n^{2} \pi^{2}}-\frac{1}{j n \pi}\right]=2\left[-\frac{2 \cos n \pi}{n \pi}+\frac{1}{n \pi}\right]=\frac{2}{\pi}\left[\frac{1}{n}(1-2 \cos n \pi)\right]=$
$\frac{2}{\pi}\left[\frac{1}{n}\left(1+2(-1)^{n+1}\right)\right] f(t)=\frac{2}{\pi}\left[3 \sin \pi t-\frac{\sin 2 \pi t}{2}+\sin 3 \pi t-\frac{\sin 4 \pi t}{4}+\ldots\right]$.

## Problem 18

The device D in the following circuit is powered by a voltage $v(t)=2+2 \cos (1000 t)+$ $\cos (2000 t)(V)$. The current across is given by $i(t)=1+\sin (1000 t)+0.5 \sin (2000 t)(A)$ find the average power associated with D .

A) 2 W
$\rightarrow$ B) -2 W
C) 4 W
D) -4 W
E) None of the above

## Problem 19

The following circuit is powered by a periodic voltage source that has the following Fourier expansion: $v(t)=21+20 \cos (1000 t)+10 \cos (2000 t)$. Find the RMS value of the current $\mathrm{i}(\mathrm{t})$.

A) 6.68 A
B) 8.14 A
C) 7.3 A
$\rightarrow$ D) 7.6 A
E) None of the above

## Problem 20

For the function $\mathrm{v}(\mathrm{t})$ given below, find the value of $a_{5}$. You may use the following:

$$
\int t \cos (\alpha t)=\frac{\cos (\alpha t)}{\alpha^{2}}+\frac{t \sin (\alpha t)}{\alpha}
$$


$\rightarrow$ A) 0
B) -0.0162
C) 1.27
D) -16.7
E) None of the above

The following given is used in the next 5 problems. 3 electrical elements are powered by a $240 V_{R M S}, 60 \mathrm{~Hz}$ source:


The following is given for the three elements:
L1: 240W, PF=0.6 Lag
L2: 200VARS, $\mathrm{PF}=0.5 \mathrm{Lag}$
L3: 100VA, $\mathrm{PF}=0$ Lead

## Problem 11

Find the total apparent power.
A) 725.67 VA
B) 626.33 VA
C) 550.2 VA
D) 888.8 VA
E) None of the above

## Problem 12

Find the total power factor.
A) 0.567 Lag
$\longrightarrow$ B) 0.646 Lag
C) 0.808 Lag
D) 0.747 Lag
E) None of the above

## Problem 13

Find the magnitude of the total current I.
A) 5.21 A
B) 3.02 A
$\rightarrow$ C) 2.292 A
D) 7.40 A
E) None of the above

## Problem 14

Find the capacitor that needs to be placed in parallel with the loads to adjust the power factor to 0.9 Lag .
A) $62.7 \mu \mathrm{~F}$
B) $147 \mu \mathrm{~F}$
C) $49 \mu \mathrm{~F}$
D) $11.4 \mu \mathrm{~F}$
E) None of the above $\mathrm{C}=2.74 \mathrm{mF}$

## Problem 15

Find the magnitude of I again after the power factor is adjusted as in the previous problem.
$\rightarrow$ A) 1.64 A
B) 3.29 A
C) 0.91 A
D) 2.13 A
E) None of the above

## Problem 10

Find X such that the maximum power transfer constraint is satisfied.

$\rightarrow$ A) $1-2.5 j \Omega$
B) $1+2.5 j \Omega$
C) $2-2.5 j \Omega$
D) $2+2.5 j \Omega$
E) None of the above

## Problem 8

Find $N_{2}$ and X such that maximum power is delivered to the $1000 \Omega$ resistor.

$\rightarrow$ A) $N_{2}=50, X=-16$
B) $N_{2}=10, X=-16$
C) $N_{2}=10, X=-0.64$
D) $N_{2}=2, X=-0.64$
E) None of the above

## Problem 3

Find R that satisfies the maximum power transfer constraint.

A) $14 \Omega$
$\rightarrow$ B) $15 \Omega$
C) $16.33 \Omega$
D) $17.46 \Omega$
E) None of the above

## Problem 2

Find the average power delivered with the 6A current source between A and B.

A) -23.86 W
$\rightarrow$ B) 23.86 W
C) 28.71 W
D) -28.71 W
E) None of the above
18. Determine $Z$ so that maximum power is transferred to it and calculate this power given that the source voltage is 10 V peak value.


Solution: We will determine TEC as seen by $Z$. On open circuit, the currents are as shown. From KVL: $10 \angle 0^{\circ}-5 \mathrm{I} / 2+5 \mathrm{I} / 2=$ $\mathbf{V}_{\mathbf{T h}}$. In This particular problem, the voltages across the $5 \Omega$ resistors cancel out. Hence, $\mathrm{V}_{\mathrm{Th}}=10 \angle 0^{\circ} \mathrm{V}$ peak value

When $Z$ is replaced by a short circuit, the currents are as shown. From
 KVL: $10 \angle 0^{\circ}-5\left(\mathbf{I}_{\text {sc }}+\mathbf{I} / 2\right)-5\left(\mathbf{I}_{\text {sc }}-\right.$ $\mathbf{I} / 2$ ) $=\mathbf{0}$. Again, the terms involving I cancel out. Hence, $I_{\text {sc }}$ $=1 \angle 0^{\circ} \mathrm{A}$, and $Z_{T h}=$ $\frac{10 \angle 0^{\circ}}{1 \angle 0^{\circ}}=10 \Omega$. It follows that for maximum power transfer, $Z=10$ $\Omega$. The power dissipated in the
 load is $\left(\frac{V_{\text {Th }}}{\sqrt{2}}\right)^{2} \frac{1}{4 \times 10}=1.25 \mathrm{~W}$.
$\frac{20}{R} R_{x}=10$, or $R_{x}=\frac{R}{2}$.
7. Determine the maximum power that can be delivered to $R_{L}$, assuming $R$ $=0.5 \Omega$.

Solution: The primary voltage of the upper transformer is always 1 V . On

open circuit, the source current is zero, the primary voltage is $5-1=4 \mathrm{~V}$, and $V_{T h}=8 \mathrm{~V}$. On short circuit, the primary voltage of the lower transformer is zero, the source current is (5 1) $/ R$ and the short circuit current is $2 / R$. This gives, $R_{T h}=4 R$. The maximum power delivered is $(8)^{2} /(4 \times 4 R)=4 / R$.
8. Given that the load $L$ consumes 1200 W at 0.8 p.f. lagging and the magnitude of the voltage across $L$ is 300 V rms. Determine the power dissipated in the resistance $R_{\text {line }}$, if $R_{\text {line }}=0.5 \Omega$.
Solution: The reactive power absorbed by the load

is $\frac{1200}{0.8} \times 0.6=900$ VAR. The reactive power absorbed by the capacitor is $\frac{V^{2}}{-30}=-3000$
VAR. The total complex power is $1200+j(900-3000)=1200-j 2100$ VA. The magnitude of
the line current $\frac{\sqrt{(1200)^{2}+(2100)^{2}}}{300}=\sqrt{65}$ A. The power dissipated in $R_{\text {line }}$ is $65 R_{\text {line }}$.
4. Determine the reactive power absorbed in the circuit, given that $I=1 \angle 0^{\circ} \mathrm{A}$ rms.
Solution: The equivalent series impedance is $\frac{j 5(5-j 5)}{j 5+5-j 5}=5+j 5$. The reactive power is $5\left|\left|\left.\right|^{2}\right.\right.$ VAR. As a

check, the current in the capacitive branch is $\frac{j 5}{j 5+5-j 5} \mathbf{I}=j \mathbf{j}$; the reactive power absorbed by the capacitor is $-\left.5|j|\right|^{2}=-\left.5| |\right|^{2}$ VAR. The current in the inductive branch is $\frac{5-j 5}{j 5+5-j 5} \mathbf{I}=(1$ $-j) \boldsymbol{I}=\sqrt{2} \angle-45^{\circ}$; the reactive power absorbed by the inductor is $5\left|\sqrt{2} \|^{2}=10\right|\left|\left.\right|^{2}\right.$ VAR. The total reactive power absorbed is $10\left|\|\left.\right|^{2}-5\right|\left|\left.\right|^{2}=5\right|\left|\left.\right|^{2}\right.$ VAR.
5. In the circuit shown, $L_{1}$ consumes 160 W at 0.8 p.f. lagging and $L_{2}$ consumes 320 VAR at 0.6 p.f. lagging. Determine $I$ when $X_{C}$ is chosen for unity power factor, assuming $\mathbf{V}_{\text {SRC }}=200 \angle 0^{\circ} \mathrm{V}$ rms.


Solution: Af unity p.f. the total reactance seen by the source is zero and the source applies only real power. The real power consumed by $L_{2}$ is $\frac{320}{0.8} \times 0.6=240 \mathrm{~W}$. The total real power supplied by the load is $160+240=400 \mathrm{~W}$. The current is $\frac{400}{V_{S R C} \angle 0^{\circ}}=\frac{400}{V_{S R C}} \angle 0^{\circ} \mathrm{A}$ rms.

8\%
5. The complex powers absorbed by $L_{1}$ and $L_{2}$ are $1+j 0.2 \mathrm{kVA}$ and $1-j 0.2 \mathrm{kVA}$. Determine $\mathbf{I}_{\mathrm{SRC}}$, assuming that the phase angle of $\mathbf{V}_{\text {SRC }}$ is zero. Note that $X$ need not be given.

A. $20 \angle 90^{\circ} \mathrm{A}$
B. $10 \angle 90^{\circ} \mathrm{A}$
C. $10 \angle 0^{\circ} \mathrm{A}$
$\rightarrow$ D. $20 \angle 0^{\circ} \mathrm{A}$
E. None of the above

Solution: The complex power delivered by the source is 2 kVA . The real power absorbed by $L_{2}$ is in the $10 \Omega$ resistor. If $\mathrm{V}_{\mathrm{SRC}}=V_{m} \angle 0 \mathrm{~V}$, then $\frac{\left|V_{m}\right|^{2}}{10}=1000$, or $V_{m}=100 \mathrm{~V}$. It follows that $I_{m}=\frac{2000}{100}=20 \mathrm{~A}$, and $\mathbf{I}_{\mathbf{S C R}}=20 \angle 0 \mathrm{~A}$.

## Problem 14



It is given that the complex power of L 1 is $5+\mathrm{j} 10 \mathrm{VA}$. It is also given that L 2 absorbs 20 W at lagging power factor of 0.8 . What is the phase difference between I and V as shown in figure?
$\rightarrow$ A) $45.00^{\circ}$
B) $39.81^{\circ}$
C) $63.33^{\circ}$
D) $60.00^{\circ}$
E) None of the above.

## Problem 15

What is the impedance of a load if it absorbs 20KVAR at lagging power factor of 0.6 when a current of magnitude 50 A rms flows through it?
$\rightarrow$ A) $6+8 \mathrm{j}$ ohms
B) $3+4 j$ ohms
C) $4+$ j3 ohms
D) $8+j 6$ ohms
E) None of the above.
4. How much complex power is delivered by the $5 \angle 30^{\circ} \mathrm{A}$ (rms) current source to the circuit shown in figure.
a. $7.5 \angle 137.48^{\circ} \mathrm{VA}$.
b. 0 VA .
c. 100 VA
$\rightarrow$ d. $15.35 \angle 137.48^{\circ} \mathrm{VA}$.
e. None of the above.


1. The values of $R_{1}, R_{2}, C$ and $L$ are unknown Load 1 absorbs a complex power of $50 \angle-45^{\circ} \mathrm{VA}$ and load 2 absorbs a complex power of $100 \angle 45^{\circ} \mathrm{VA}$. Determine $R_{2}$ if $V_{s}=250 \angle 0^{\circ}$ Vrms.
a. $250 \sqrt{2} \Omega$
b. $125 \sqrt{2} \Omega$
c. $125 \Omega$
d. $125 / \sqrt{2} \Omega$.
e. None of the above.

2. Find the voltage $v_{0}(t)$ across the capacitor of the circuit shown in figure.

3. Two impedances $Z_{1}=9.8 \angle-78^{\circ} \Omega$ and $Z_{2}=18.5 \angle 21.8^{\circ} \Omega$ are connected in parallel and the combination in series with an impedance $Z_{3}=5 \angle 53^{\circ}$. If this circuit is connected across a $100-\mathrm{V}$ source (ms), how much average power will be supplied by the source.
$\rightarrow$ a. 980.8 W .
b. 490 W .
c. 1960 W .
d. 1391.6 W.
e. None of the above.
4. An impedance $Z 1=(4+\mathrm{j} 4) \Omega$ is connected in parallel with an impedance $Z 2=$ $(12+j 6) \Omega$. If the input reactive power is 1000 VAR (lagging), what is the total active (average) power?
A. 1210 W
B. 3025 W
C. 826.39 W
D. 1150 W
E. None of the above
5. The conjugate of the complex power delivered by a current source is $200-j 200$ the source.
A. 40 Vrms
B. $j 40 \mathrm{Vrms}$
C. 80 Vrms
D. $-j 40 \mathrm{Vrms}$
$\rightarrow$ E. None of the above j40sqrt(2) rms
6. Determine the value of C in the circuit shown if C takes 5 VAR . The operating frequency is 50 Hz .
A. $12.63 \mu \mathrm{~F}$
B. $14.74 \mu \mathrm{~F}$
C. $17.68 \mu \mathrm{~F}$
D. $3 \mu \mathrm{~F}$

E. None of the above
7. In the circuit shown, the capacitance absorbs - 200 VAR. Determine the average power dissipated in $R$ if $R=5 \Omega$.
A. 57.1 W
B. 80 W
C. 44.4 W
D. 66.7 W
E. 50 W


Solution: $Q=-B V_{\mathrm{rms}}^{2}$, where $V_{\mathrm{rms}}$ is the rms voltage across $R$ and $C$, and $B=-1 / X=1 / 2 \mathrm{~S}$. Substituting, $-200=-\frac{1}{2} V_{\mathrm{rms}}^{2}$, and $V_{\mathrm{rms}}=20 \mathrm{~V}$. It follows that $P_{R}=\frac{V_{\mathrm{rms}}^{2}}{R}=\frac{400}{5}=80 \mathrm{~W}$.
3. Determine $\mathbf{I}_{\mathrm{x}}$ assuming $\mathbf{I}_{\mathrm{SRC}}=j \mathrm{~A}$.
A. $j 6 \mathrm{~A}$
B. $-j 3 \mathrm{~A}$
C. $j 3 \mathrm{~A}$
D. $-j 6 \mathrm{~A}$
E. $j 4 \mathrm{~A}$

Solution: The voltage across the $-j 3 \Omega$ capacitor is 6 V and the current through this capacitor, directed upwards is $j 2 \mathrm{~A}$. It
 follows that $\mathbf{I}_{\mathbf{x}}=\mathbf{I}_{\text {SRC }}+j 2=j 3 \mathrm{~A}$.
5. Two coils are tightly coupled to a high-permeability core, so that the leakage flux is negligibly small. If coil 1 has 100 turns and an inductance of 10 mH , and the mutual inductance is 12.5 mH , determine the number of turns of coil 2 .
A. 125
B. 250
C. 150
D. 175
E. 200

Solution: From the definitions of self and mutual inductance, with negligible leakage flux,
$L_{1}=\frac{N_{1} \phi_{21}}{i_{1}}$ and $M=\frac{N_{2} \phi_{21}}{i_{1}}$. It follows that $N_{2}=\frac{M}{L_{1}} N_{1}=10 M=125$.
6. Determine the inductance of coil 2 of the preceding problem.
A. 22.5 mH
B. 30.63 mH
C. 15.63 mH
D. 40 mH
E. 50.63 mH

Solution: Since the coils are tightly coupled to the core, $k=1$, so that $M^{2}=L_{1} L_{2}$, or $L_{2}=\frac{M^{2}}{L_{1}}=0.1 M^{2} \mathrm{mH}$. It also follows from the solution of the preceding problem that $N_{1}=\frac{M}{L_{2}} N_{2}$. Dividing, $L_{2}=L_{1}\left(\frac{N_{2}}{N_{1}}\right)^{2}=0.1 M^{2}=0.1 \times(12.5)^{2}=15.625 \mathrm{mH}$.
7. A D'Arsonval movement has a resistance of $R \Omega$ and a full-scale deflection of $100 \mu \mathrm{~A}$. Determine the shunt resistance that will result in a full-scale deflection of $150 \mu \mathrm{~A}$, assuming $R=50 \Omega$.
A. $150 \Omega$
B. $200 \Omega$
C. $300 \Omega$
D. $100 \Omega$
E. $250 \Omega$

Solution: At full-scale deflection, the voltage drop across the movement and shunt is ( $R$ $\Omega) \times(100 \mu \mathrm{~A})=100 R \mu \mathrm{~V}$. The shunt has to pass $50 \mu \mathrm{~A}$, so its resistance is $R_{\text {shunt }}=100 R / 50=$ $2 R=100 \Omega$.
8. When a $9950 \Omega$ resistance is connected in series with a D'Arsonval movement of unknown resistance and full-scale deflection current, a voltage of 1 V across the series combination gives a certain full-scale deflection. If an additional $10,000 \Omega$ is connected in series with the combination, 2 V are required for full-scale deflection. Determine the resistance of the D'Arsonval movement.
A. $150 \Omega$
B. $100 \Omega$
C. $75 \Omega$
D. $125 \Omega$
E. $50 \Omega$

Solution: Let the resistance of the movement be $R_{m}$, its FSD current be $I_{\text {FSD }}$, and the FSD voltage with series resistance be $V_{\text {FSD }}$. Then $I_{\text {FSD }}\left(R+R_{m}\right)=V_{\text {FSD }}$, and $I_{\text {FSD }}(10,000+R+$ $\left.R_{m}\right)=2 V_{\text {FSD }}$. It follows that $R+R_{m}=10,000$, or $R_{m}=10,000-R=50 \Omega$.
9. Determine $L_{e q}$ if $L=1 \mathrm{H}$.
A. 6 H
B. 4 H
C. 8 H
D. 7 H

E. 5 H

Solution: Consider that a voltage $\mathbf{V}$ is applied, causing a current $\mathbf{I}$ to flow. $\mathbf{V}=j \omega \mathbf{l}[(2-1+1)$ $+(3-1-1)+(L+1-1)] ; L_{e q}=3+L=4 \mathrm{H}$.
10. Determine $\mathbf{V}_{\mathbf{T h}}$, assuming $\mathbf{V}_{\mathbf{S R C}}=1 \angle 0^{\circ} \mathrm{V}$
A. $-1 \angle 0^{\circ} \mathrm{V}$
B. $1 \angle 0^{\circ} \mathrm{V}$
C. $-2 \angle 0^{\circ} \mathrm{V}$
D. $2 \angle 0^{\circ} \mathrm{V}$
E. $4 \angle 0^{\circ} \mathrm{V}$

Solution: On open circuit, no current flows. The primary voltage is $\mathrm{V}_{\mathrm{SRC}}$ as shown, and $\mathrm{V}_{\mathrm{Th}}=-\mathrm{V}_{\mathrm{SRC}}=-1 \angle 0^{\circ} \mathrm{V}$
11. Determine $Z_{L}$ for maximum average power delivered to it if $R=5 \Omega$ and $\mathbf{I}_{\mathrm{x}}=k \angle-45^{\circ}$ where $k=\sqrt{2}$ Arms.
A. $10+j 10 \Omega$
B. $5+j 5 \Omega$
C. $5-j 5 \Omega$
D. $10-j 10 \Omega$
E. $15-j 15 \Omega$


Solution: $Z_{T h}$ is $(R+j 5) \Omega$. Hence, $Z_{L}$ for maximum power transfer is $(R-j 5)=(5-j 5) \Omega$.
12. Determine the maximum average power delivered to $Z_{L}$ in Problem 11, assuming that $R$ $=5 \Omega$ and $\mathrm{I}_{\mathrm{x}}$ is as in Problem 11.
A. 90 W
B. 200 W
C. 320 W
D. 180 W
E. 245 W

Solution: $V_{T h}$ as seen by $Z_{L}$ is determined from superposition as $\frac{j 10}{j 10+j 10} \times 100 \angle 0^{\circ}+$ $(5+j 10 \| j 10) \mathrm{I}_{\mathrm{x}}=50 \angle 0^{\circ}+5(1+j) \mathrm{I}_{\mathrm{x}}=50 \angle 0^{\circ}+\left(5 \sqrt{2} \angle 45^{\circ}\right) \times k \angle-45^{\circ}=50+5 \sqrt{2} k$; $P_{L \text { max }}=\frac{(50+5 \sqrt{2} k)^{2}}{4 R_{T h}}=\frac{(50+5 \sqrt{2} k)^{2}}{20}=\frac{(50+5 \sqrt{2} \times \sqrt{2})^{2}}{20}=\frac{(60)^{2}}{20}=180 \mathrm{~W}$.
13. Load $L_{1}$ absorbs 15 kVA at 0.6 p.f. lagging, whereas Load $L_{2}$ absorbs 4.8 kW at 0.8 p.f. leading. If $\mathbf{V}_{\mathrm{SRC}}=200 \angle 0^{\circ} \mathrm{V} \mathrm{rms}$ at $f=50 \mathrm{~Hz}$, determine the capacitor that must be connected in parallel with $L_{1}$ and $L_{2}$ to have maximum
 magnitude of current through the source.
A. 0.67 mF
B. 0.55 mF
C. 0.34 mF
D. 0.46 mF
E. 1.24 mF

Solution: The reactive power absorbed by $L_{1}$ is $15 \times 0.8 \mathrm{kVAR}=12 \mathrm{kVAR}$, whereas the reactive power absorbed by $L_{2}$ is $-\frac{4.8}{0.8} \times 0.6=-3.6 \mathrm{kVAR}$. For maximum magnitude of source current, the p.f. should be unity. The capacitor must add a reactive power of -(12 3.6) $=-8400$ VAR. hence, $-8400=-\omega C \times\left|\mathrm{V}_{\mathrm{SRC}}\right|^{2}$, or $C=\frac{8400}{100 \pi\left|\mathrm{~V}_{\mathrm{SRC}}\right|^{2}}=\frac{84}{\pi\left|\mathrm{~V}_{\mathrm{SRC}}\right|^{2}}=$ $\frac{84}{\pi(200)^{2}} \equiv 0.67 \mathrm{mF}$.
14. A periodic current is shown, where over a period,

$$
\begin{array}{lr}
i=6+A \sin 2 t & 0 \leq \mathrm{t} \leq \pi \\
i=-4+A \sin 2(t-\pi) \quad \pi 0 \leq \mathrm{t} \leq 2 \pi
\end{array}
$$

Determine the rms value of $i$ if $A=1 \mathrm{~A}$.
A. 5.83 A
B. 5.15 A

C. 6.20 A
D. 5.29 A
E. 5.52 A

Solution: The waveform consists of three components: i) a dc component of 1 A, ii) a square wave of 5 V amplitude, and iii) a sinusoidal wave of amplitude $A$. It follows that the rms value is $I=\sqrt{1^{2}+5^{2}+A^{2} / 2}=\sqrt{26+A^{2} / 2}=\sqrt{26.5}=5.15 \mathrm{~A}$.
15. The current waveform of the preceding problem is applied to a $2 \Omega$ resistor in parallel with a very large capacitor. Determine the voltage across the parallel combination.
A. 2.5 V
B. 2 V
C. 3 V
D. 4 V
E. 3.5 V

Solution: The ac voltage will be negligibly small. The dc voltage is the dc component of current multiplied by $R$, or $V=1 \times R=2 \mathrm{~V}$.
16. The period of a periodic function $f(t)$ is defined as:

$$
\begin{array}{ll}
f(t)=\cos (t+\pi)-2, & -\pi<t<-\pi / 2 \\
f(t)=-\cos (t)+k, & -\pi / 2<t<+\pi / 2 \\
f(t)=\cos (t-\pi)-2, & \pi / 2<t<\pi
\end{array}
$$

Derive the trigonometric Fourier series expansion of $f(t)$, assuming $k=3$.


Solution: $a_{o}=\frac{1}{\pi}\left[\int_{0}^{\pi / 2}(-\cos t+k) d t+\int_{\pi / 2}^{\pi}(\cos (t-\pi)-2) d t\right]=$
$\frac{1}{\pi}\left[k \int_{0}^{\pi / 2} d t-2 \int_{\pi / 2}^{\pi} d t-\int_{0}^{\pi / 2} \cos t d t-\int_{\pi / 2}^{\pi} \cos t d t\right]=\frac{1}{\pi}\left[k \int_{0}^{\pi / 2} d t-2 \int_{\pi / 2}^{\pi} d t\right]=\frac{1}{\pi}\left[\frac{k \pi}{2}-\pi\right]=\frac{k}{2}-1$.
$a_{n}=\frac{2}{\pi}\left[\int_{0}^{\pi / 2}(-\cos t+k) \cos n t d t+\int_{\pi / 2}^{\pi}(\cos (t-\pi)-2) \cos n t d t\right]=$
$\frac{2}{\pi}\left[-\int_{0}^{\pi} \cos t \cos n t d t+\int_{0}^{\pi / 2} k \cos n t d t-\int_{\pi / 2}^{\pi} 2 \cos n t d t\right]=$
$\frac{2}{\pi}\left[-\frac{1}{2} \int_{0}^{\pi} \cos (n-1) t d t-\frac{1}{2} \int_{0}^{\pi} \cos (n+1) t d t+k \int_{0}^{\pi / 2} \cos n t d t-2 \int_{\pi / 2}^{\pi} \cos n t d t\right]=$
$-\frac{1}{\pi}\left[\frac{\sin (n-1) t}{n-1}+\frac{\sin (n+1) t}{n+1}\right]_{0}^{\pi}+\frac{2 k}{n \pi}[\sin n t]_{0}^{\pi / 2}-\frac{4}{n \pi}[\sin n t]_{\pi / 2}^{\pi}=$
$0-0+\frac{2 k}{n \pi} \sin \frac{n \pi}{2}-0-0+\frac{4}{n \pi} \sin \frac{n \pi}{2}=\frac{2(k+2)}{n \pi} \sin \frac{n \pi}{2} . a_{n}$ is zero for even values, and the odd harmonics alternate in sign. Thus,

$$
\begin{aligned}
& f(t)=\left(\frac{k}{2}-1\right) \frac{1}{2}+\frac{2(k+2)}{\pi}\left(\cos t-\frac{1}{3} \cos 3 t+\frac{1}{5} \cos 5 t-\frac{1}{7} \cos 7 t+\ldots\right) \\
& =\frac{1}{2}+\frac{10}{\pi}\left(\cos t-\frac{1}{3} \cos 3 t+\frac{1}{5} \cos 5 t-\frac{1}{7} \cos 7 t+\ldots\right) .
\end{aligned}
$$

19. Determine $X$ and $R$ for maximum power transfer to $R$ and calculate this power. Assume $\mathrm{V}_{\text {SRC }}=4 \angle 0^{\circ} \mathrm{V} \mathrm{rms}$.


Solution: On open circuit, $\mathrm{V}_{\mathrm{TH}}=\mathrm{V}=\mathrm{V}_{\mathrm{SRC}}$. On short circuit, $\mathbf{V}=0$ and $\mathbf{I}_{\mathrm{N}}=\mathbf{I}=\frac{\mathbf{V}_{\text {SRC }}}{5(1+j)} . Y_{N}=$ $\frac{1}{5(1+j)}=0.1(1-j) \mathrm{S}$. For maximum power transfer, $G_{L}=0.1 \mathrm{~S}$, or $R=10 \Omega$, and $B_{L}=0.1$ S , or $X=-10 \Omega$.


Under conditions of maximum power transfer, the current in $R$ is $0.5\left|I_{N}\right|=\frac{0.5\left|\mathrm{~V}_{\mathrm{SRC}}\right|}{10}$ and the power transferred is $\frac{0.25\left|\mathbf{V}_{\mathrm{SRC}}\right|^{2}}{10}=$ $\frac{\left|\mathrm{V}_{\mathrm{SRC}}\right|^{2}}{40}=\frac{16}{40}=0.4 \mathrm{~W}$.

20. Determine the complex power delivered by each source given that $\mathbf{V}_{\text {SRC }}=5 \cos \omega t$, $\mathbf{I S R C}=-2 \sin \omega t$, and assuming $Z_{L}=k(1-j)$ where $k=1$.
Solution: The currents and voltages are as shown.
Equating mmfs: $100 \times 2 \angle 90^{\circ}+200 \mathrm{I}_{\mathrm{L}}=0$, or $j 2=-2 \mathrm{~L}_{\mathrm{L}}$, and $\mathbf{I}_{\mathrm{L}}=-j \mathrm{~A}, \mathbf{I}_{\mathbf{I}}=\mathbf{I}_{\mathrm{L}}-j 2=-j 3 \mathrm{~A}$.
$\mathbf{V}_{\mathbf{L}}=Z_{\mathbf{L}} \mathbf{I}_{\mathbf{L}}=-j k(1-j)=-k(1+j) \mathrm{V} . \mathbf{V}_{\mathbf{2}}=\mathbf{V}_{\mathbf{L}}-5=-(k+$
5) $-j k . \mathbf{V}_{1}=\mathbf{V}_{2} / 2=-(k+5) / 2-j k / 2 \mathrm{~V} . \mathbf{V}_{\mathbf{1}}=5-\mathbf{V}_{1}=$ $(15+k) / 2+j k / 2 \mathrm{~V}$.
Power delivered by voltage source $=S_{v}=$

$\frac{V_{s r c}}{\sqrt{2}} \frac{I_{1}^{*}}{\sqrt{2}}=\frac{1}{2}(5)(j 3)=\frac{j 15}{2}=j 7.5 \mathrm{VA}$
Power delivered by current source $S_{1}=$
$\frac{\mathrm{V}_{1}}{\sqrt{2}} \frac{\mathrm{I}_{\mathrm{SRC}}^{*}}{\sqrt{2}}=\frac{1}{2}\left(\frac{15+k}{2}+\frac{j k}{2}\right)(-j 2)=$
$\frac{1}{2}(k-j(15+k))=0.5-j 8 \mathrm{VA}$
Total power delivered by sources $=$
$\left.\frac{1}{2}(j 15+k-j 15-j k)\right)=\frac{k}{2}(1-j)$
As a check, $S_{L}=$
$\frac{V_{L m}}{\sqrt{2}} \frac{I_{L m}^{*}}{\sqrt{2}}=\frac{1}{2}(-j k(1-j))(j)=\frac{k}{2}(1-j) \mathrm{VA}$.

2. In the circuit shown, the capacitance absorbs - 200 VAR. Determine the average power dissipated in $R$ if $R=5 \Omega$.

Solution: $Q=-B V_{\mathrm{rms}}^{2}$, where $V_{\mathrm{rms}}$ is the rms voltage across $R$ and $C$, and $B=-1 / X=1 / 2 \mathrm{~S}$. Substituting, $-200=-\frac{1}{2} V_{\mathrm{rms}}^{2}$, and $V_{\mathrm{rms}}=20 \mathrm{~V}$. It
 follows that $P_{R}=\frac{V_{\text {ris }}^{2}}{R}=\frac{400}{5}=80 \mathrm{~W}$.
8. When a $9950 \Omega$ resistance is connected in series with a D'Arsonval movement of unknown resistance and full-scale deflection current, a voltage of 1 V across the series combination gives a certain full-scale deflection. If an additional $10,000 \Omega$ is connected in series with the combination, 2 V are required for full-scale deflection. Determine the resistance of the D'Arsonval movement.
Solution: Let the resistance of the movement be $R_{m}$, its FSD current be $I_{\text {FSD }}$, and the FSD voltage with series resistance be $V_{\text {FSD }}$. Then $I_{\text {FSD }}\left(R+R_{m}\right)=V_{\text {FSD }}$, and $I_{\text {FSD }}(10,000+R+$ $\left.R_{m}\right)=2 V_{\text {FSD }}$. It follows that $R+R_{m}=$ 10,000 , or $R_{m}=10,000-R=50 \Omega$.
11. Determine $Z_{L}$ for maximum average power delivered to it if $R=5 \Omega$ and $\mathrm{I}_{\mathrm{x}}=k \angle-45^{\circ}$ where $k=\sqrt{2} \mathrm{Arms}$.
Solution: $Z_{T h}$ is $(R+j 5) \Omega$. Hence, $Z_{L}$ for maximum power transfer is $(R-j 5)=(5-j 5)$ $\Omega$.

12. Determine the maximum average power delivered to $Z_{L}$ in Problem 11, assuming that $R$ $=5 \Omega$ and $\mathrm{I}_{\mathrm{x}}$ is as in Problem 11.
Solution: $\mathrm{V}_{\mathrm{Th}}$ as seen by $Z_{L}$ is determined from superposition as $\frac{j 10}{j 10+j 10} \times 100 \angle 0^{\circ}+$ $(5+j 10 \| j 10) \mathrm{I}_{\mathrm{x}}=50 \angle 0^{\circ}+5(1+j) \mathrm{I}_{\mathrm{x}}=50 \angle 0^{\circ}+\left(5 \sqrt{2} \angle 45^{\circ}\right) \times k \angle-45^{\circ}=50+5 \sqrt{2} k ;$
$P_{L \max }=\frac{(50+5 \sqrt{2} k)^{2}}{4 R_{T h}}=\frac{(50+5 \sqrt{2} k)^{2}}{20}=\frac{(50+5 \sqrt{2} \times \sqrt{2})^{2}}{20}=\frac{(60)^{2}}{20}=180 \mathrm{~W}$.
13. Load $L_{1}$ absorbs 15 kVA at 0.6 p.f. lagging, whereas Load $L_{2}$ absorbs 4.8 kW at 0.8 p.f. leading. If $\mathbf{V}_{\mathbf{S R C}}=200 \angle 0^{\circ} \mathrm{V} \mathrm{rms} \mathrm{at} f=50 \mathrm{~Hz}$, determine the capacitor that must be connected in parallel with $L_{1}$ and $L_{2}$ to have maximum
 magnitude of current through the source.

Solution: The reactive power absorbed by $L_{1}$ is $15 \times 0.8 \mathrm{kVAR}=12 \mathrm{kVAR}$, whereas the reactive power absorbed by $L_{2}$ is $-\frac{4.8}{0.8} \times 0.6=-3.6 \mathrm{kVAR}$. For maximum magnitude of source current, the p.f. should be unity. The capacitor must add a reactive power of -(12 3.6) $=-8400$ VAR. hence, $-8400=-\omega C \times\left|\mathbf{V}_{\mathrm{SRC}}\right|^{2}$, or $C=\frac{8400}{100 \pi\left|\mathrm{~V}_{\mathrm{SRC}}\right|^{2}}=\frac{84}{\pi\left|\mathrm{~V}_{\mathrm{SRC}}\right|^{2}}=$ $84 /(\pi 2002) \equiv 0.67 \mathrm{mF}$.
20. Determine the complex power delivered by each source given that $\mathbf{V}_{\mathbf{S R C}}=5 \cos \omega t$, $\mathbf{I}_{\mathbf{S R C}}=-2 \sin \omega t$, and assuming $Z_{L}=k(1-j)$ where $k=1$.

Solution: The currents and voltages are as shown.
Equating mmfs: $100 \times 2 \angle 90^{\circ}+200 \mathrm{I}_{\mathrm{L}}=0$, or $j 2=-2 \mathrm{I}_{\mathrm{L}}$, and $I_{L}=-j A, I_{1}=I_{L}-j 2=-j 3 A$.
$\mathbf{V}_{\mathbf{L}}=Z_{\mathrm{L}} \mathbf{I}_{\mathbf{L}}=-j k(1-j)=-k(1+j) \mathrm{V} . \mathrm{V}_{\mathbf{2}}=\mathrm{V}_{\mathrm{L}}-5=-(k+$ 5) $-j k . \mathbf{V}_{1}=\mathrm{V}_{2} / 2=-(k+5) / 2-j k / 2 \mathrm{~V} . \mathrm{V}_{\mathbf{1}}=5-\mathrm{V}_{1}=$ $(15+k) / 2+j k / 2 \mathrm{~V}$.
Power delivered by voltage source $=S_{v}=$

$\frac{V_{s r c}}{\sqrt{2}} \frac{I_{1}^{*}}{\sqrt{2}}=\frac{1}{2}(5)(j 3)=\frac{j 15}{2}=j 7.5 \mathrm{VA}$
Power delivered by current source $S_{I}=$
$\frac{\mathrm{V}_{1}}{\sqrt{2}} \frac{\mathrm{I}_{\mathrm{SRC}}^{*}}{\sqrt{2}}=\frac{1}{2}\left(\frac{15+k}{2}+\frac{j k}{2}\right)(-j 2)=$
$\frac{1}{2}(k-j(15+k))=0.5-j 8 \mathrm{VA}$
Total power delivered by sources =
$\left.\frac{1}{2}(j 15+k-j 15-j k)\right)=\frac{k}{2}(1-j)$
As a check, $S_{L}=$
$\frac{V_{L m}}{\sqrt{2}} \frac{I_{L m}^{*}}{\sqrt{2}}=\frac{1}{2}(-j k(1-j))(j)=\frac{k}{2}(1-j) \mathrm{VA}$.

15. The current waveform of the preceding problem is applied to a $2 \Omega$ resistor in parallel with a very large capacitor. Determine the voltage across the parallel combination.

Solution: The ac voltage will be negligibly small. The dc voltage is the dc component of current multiplied by $R$, or $V=1 \times R=2 \mathrm{~V}$.
16. The period of a periodic function $f(t)$ is defined as:

$$
\begin{array}{ll}
f(t)=\cos (t+\pi)-2, & -\pi<t<-\pi / 2 \\
f(t)=-\cos (t)+k, & -\pi / 2<t<+\pi / 2 \\
f(t)=\cos (t-\pi)-2, & \pi / 2<t<\pi
\end{array}
$$

Derive the trigonometric Fourier series expansion of $f(t)$, assuming $k=3$.


Solution: $a_{o}=\frac{1}{\pi}\left[\int_{0}^{\pi / 2}(-\cos t+k) d t+\int_{\pi / 2}^{\pi}(\cos (t-\pi)-2) d t\right]=$
$\frac{1}{\pi}\left[k \int_{0}^{\pi / 2} d t-2 \int_{\pi / 2}^{\pi} d t-\int_{0}^{\pi / 2} \cos t d t-\int_{\pi / 2}^{\pi} \cos t d t\right]=\frac{1}{\pi}\left[k \int_{0}^{\pi / 2} d t-2 \int_{\pi / 2}^{\pi} d t\right]=\frac{1}{\pi}\left[\frac{k \pi}{2}-\pi\right]=\frac{k}{2}-1$.
$a_{n}=\frac{2}{\pi}\left[\int_{0}^{\pi / 2}(-\cos t+k) \cos n t d t+\int_{\pi / 2}^{\pi}(\cos (t-\pi)-2) \cos n t d t\right]=$
$\frac{2}{\pi}\left[-\int_{0}^{\pi} \cos t \cos n t d t+\int_{0}^{\pi / 2} k \cos n t d t-\int_{\pi / 2}^{\pi} 2 \cos n t d t\right]=$
$\frac{2}{\pi}\left[-\frac{1}{2} \int_{0}^{\pi} \cos (n-1) t d t-\frac{1}{2} \int_{0}^{\pi} \cos (n+1) t d t+k \int_{0}^{\pi / 2} \cos n t d t-2 \int_{\pi / 2}^{\pi} \cos n t d t\right]=$
$-\frac{1}{\pi}\left[\frac{\sin (n-1) t}{n-1}+\frac{\sin (n+1) t}{n+1}\right]_{0}^{\pi}+\frac{2 k}{n \pi}[\sin n t]_{0}^{\pi / 2}-\frac{4}{n \pi}[\sin n t]_{\pi / 2}^{\pi}=$
$0-0+\frac{2 k}{n \pi} \sin \frac{n \pi}{2}-0-0+\frac{4}{n \pi} \sin \frac{n \pi}{2}=\frac{2(k+2)}{n \pi} \sin \frac{n \pi}{2} . a_{n}$ is zero for even values, and the odd harmonics alternate in sign. Thus,
$f(t)=\left(\frac{k}{2}-1\right) \frac{1}{2}+\frac{2(k+2)}{\pi}\left(\cos t-\frac{1}{3} \cos 3 t+\frac{1}{5} \cos 5 t-\frac{1}{7} \cos 7 t+\ldots\right)$
$=\frac{1}{2}+\frac{10}{\pi}\left(\cos t-\frac{1}{3} \cos 3 t+\frac{1}{5} \cos 5 t-\frac{1}{7} \cos 7 t+\ldots\right)$.
7. A D'Arsonval movement has a resistance of $R \Omega$ and a full-scale deflection of $100 \mu \mathrm{~A}$. Determine the shunt resistance that will result in a full-scale deflection of $150 \mu \mathrm{~A}$, assuming $R=50 \Omega$.

Solution: At full-scale deflection, the voltage drop across the movement and shunt is ( $R$ $\Omega) \times(100 \mu \mathrm{~A})=100 \mathrm{R} \mu \mathrm{V}$. The shunt has to pass $50 \mu \mathrm{~A}$, so its resistance is $R_{\text {shunt }}=100 R / 50=$ $2 R=100 \Omega$.
19. Determine $X$ and $R$ for maximum power


Solution: On open circuit, $\mathbf{V}_{\mathrm{TH}}=\mathbf{V}=\mathbf{V}_{\mathrm{SRc}}$. On short circuit, $\mathbf{V}=0$ and $\mathbf{I}_{\mathrm{N}}=\mathbf{I}=\frac{\mathbf{V}_{\text {SRC }}}{5(1+j)} \cdot Y_{N}=$ $\frac{1}{5(1+j)}=0.1(1-j) \mathrm{S}$. For maximum power transfer, $G_{L}=0.1 \mathrm{~S}$, or $R=10 \Omega$, and $B_{L}=0.1$ S , or $X=-10 \Omega$.

Under conditions of maximum power transfer, the current in $R$ is $0.5\left|I_{\mathrm{N}}\right|=\frac{0.5\left|\mathrm{~V}_{\text {SRC }}\right|}{10}$ and the power transferred is $\frac{0.25\left|\mathbf{V}_{\text {SRC }}\right|^{2}}{10}=$ $\frac{\left|\mathrm{V}_{\mathrm{SRC}}\right|^{2}}{40}=\frac{16}{40}=0.4 \mathrm{~W}$.
14. A periodic current is shown, where over a period,

$$
\begin{array}{lr}
i=6+A \sin 2 t \quad 0 \leq \mathrm{t} \leq \pi \\
i=-4+A \sin 2(t-\pi) & \pi 0 \leq \mathrm{t} \leq 2 \pi
\end{array}
$$

Determine the rms value of $i$ if $A=1 \mathrm{~A}$.
Solution: The waveform consists of three components: i) a dc component of 1 A , ii) a
 rms.
 square wave of 5 V amplitude, and iii) a sinusoidal wave of amplitude $A$. It follows that the rms value is $I=\sqrt{1^{2}+5^{2}+A^{2} / 2}=\sqrt{26+A^{2} / 2}=\sqrt{26.5}=5.15 \mathrm{~A}$.

